UNIT – 1

(Electrical technology)

Introduction:-

→ Electricity plays an important role in our day-to day life.

→ In fact, now-a-days, all our activities are totally dependent upon electricity.

Defination of Electricity:-

→ The invisible energy which constitutes the flow of electrons in a closed circuit to do work is called electricity.

→ It is a form of energy which can be converted to any other form very easily.

Electric Potential:-

⇒ The capacity of a charged body to do work is called electric potential

→ In other words, we can say that:-

The capacity of a charged body to do work determines the electric potential on it

Electric potential = $\frac{\text{Work done}}{\text{Charge}}$ or $V = \frac{W}{Q}$

Unit → $W = 1$ joules , $Q = 1$ Coulomb

$V = 1/1 = 1$ Volt

Potential Difference :-

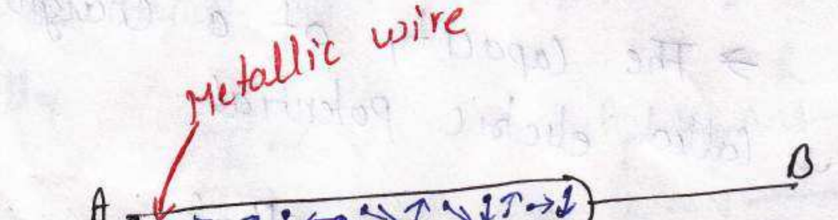
The difference in the electric potential of the two charged bodies is called potential difference.

Unit :- The unit of potential difference is Volt.

Electric Current :-

→ A continuous flow of electrons in an electric circuit is called electric current.

→ In conducting materials, a large number of free electron are available which moves from one atom to the other at random as shown in fig ①



(fig - ①)

→ When an electric potential difference is applied across the metallic wire, the loosely attached free electrons start moving towards the +ve terminal of the cell. This continuous flow of electrons constitutes

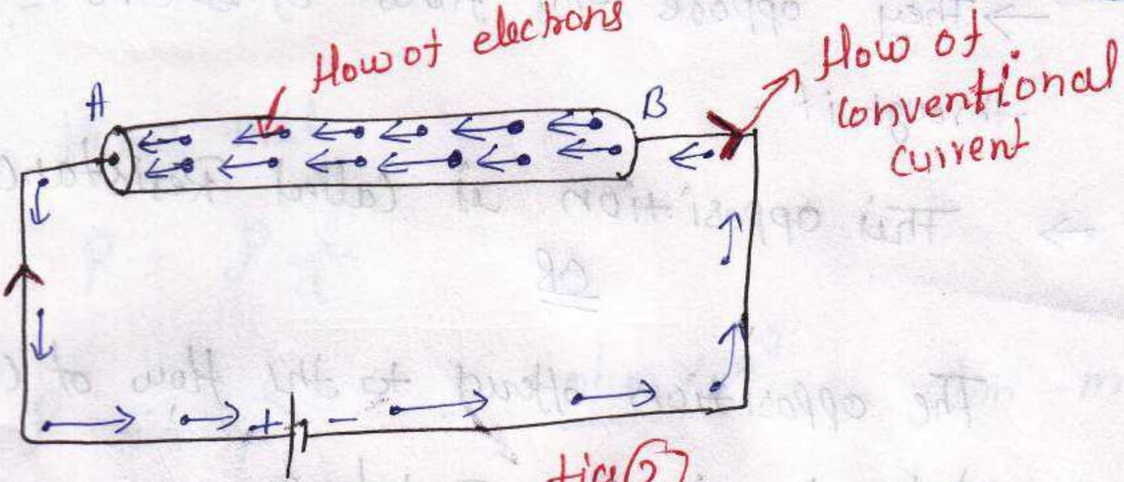


fig (2)

Mathematically ;

Current, $I = Q/t = \frac{\text{Charge}}{\text{time}}$

Unit $I = \frac{\text{Coulomb}}{\text{second}}$

I = Ampere (A)

Resistance:-

→ When a potential difference is applied across a conductor (or wire), the free electrons start moving in a particular direction. While moving through the material, these electrons collide with other atoms and molecules (see fig (3))

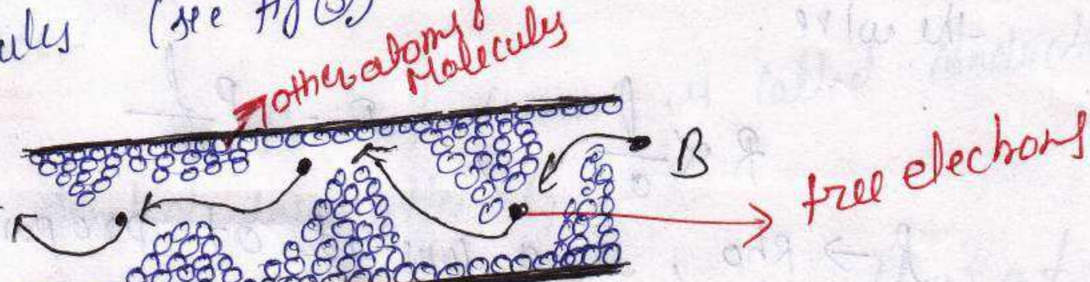


fig (3)

→ They oppose this flow of electrons (or current) through it.

→ This opposition is called Resistance.

OR

The opposition offered to the flow of current (free electrons) is called Resistance.

Unit → ohm or kilo-ohms

Symbol → Ω or $K\Omega$

⊗ **Laws of Resistance:-**

→ the Resistance (R) of a wire depends upon the various factors.

1) $R \propto l$; l → length

2) $R \propto \frac{l}{a}$; a → area of cross-section

3) It depends upon the nature (i.e. atomic structure) of the material of which the wire is made

4) It also depends upon the temperature of the wire.

$$R \propto \frac{l}{a} \; ; \; R = \rho \frac{l}{a}$$

ρ → Rho, is a constant of proportionality called Resistivity of the wire material. Its value depends upon the third factor above.

⊗ Resistivity :-

We know that

$$R = \rho \frac{l}{a}$$

$$\rho = \frac{Ra}{l} = \frac{ohm \times m^2}{m} = ohm\text{-}m$$

Unit → ohm-m

⊗ Specific Resistance :-

→ The Resistivity of the material is also known as specific Resistance of that material.

→ Because it represents the Resistance of a material having specific dimensions.

Defination :-

→ specific Resistance of a material is defined as the Resistance of that material having specific dimensions.

⊗ Conductance :-

the flow of current is called conductance.

→ It is denoted by letter G

We know that, the opposition to the flow of current is called Resistance.

Hence the conductance is just Reciprocal of Resistance

$$G = \frac{1}{R} = \frac{1 \times a}{\rho l} = \sigma \frac{a}{l}$$

Unit = mho (i.e ohm spelt backward)

symbol → Ω

σ → conductivity or specific conductance of that material.

$$G = \sigma \frac{a}{l} \quad \text{or} \quad \sigma = G \cdot \frac{l}{a}$$

$$\sigma = \frac{mho \times m}{m^2} = mho/meter \rightarrow unit$$

→ It is basically the nature of the material due to which it allows the current to flow through it.

⊗ EMF :- (electro motive force)

→ The e.m.f of a source (say a battery) is a measure of the energy that it gives to each Coulomb of charge.

→ At first sight, the name e.m.f implies that it is a force that causes the electrons (the charged particle i.e current) to flow through the circuit

In fact, it is not a force but it is energy supplied by some active source such as battery to one coulomb of charge.

(*) E.M.F & Potential difference :-

→ The amount of energy supplied by the source to each coulomb of charge is known as e.m.f of the source

→ whereas, the amount of energy used by one coulomb of charge in moving from one point to the other is known as potential difference between the two points.

example :-

for instant, consider a ckt shown in fig (4)

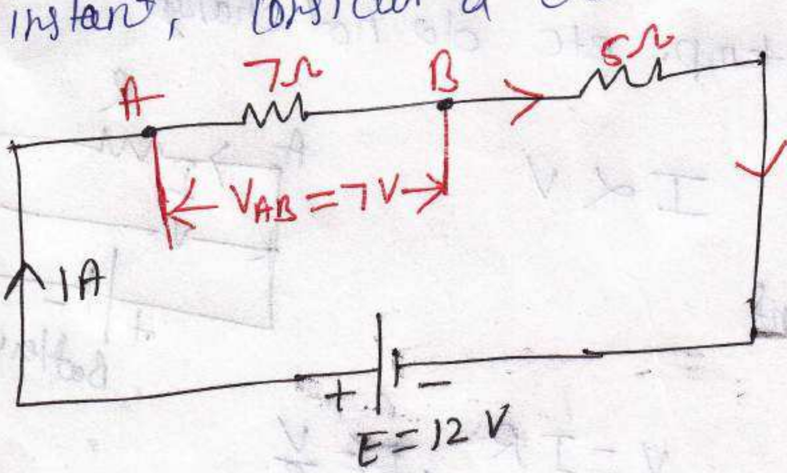


fig (4)

→ A battery has an e.m.f 12 V, it means that the battery supplies 12 Joules of energy to each coulomb of charge continuously.

→ when each coulomb of charge travels from +ve terminal to -ve terminal through external circuit, it gives up whole of the energy originally supplied by the battery.

→ The potential difference b/w any two points say A & B, is the energy used by one coulomb of charge in moving from one point (A) to the other B.

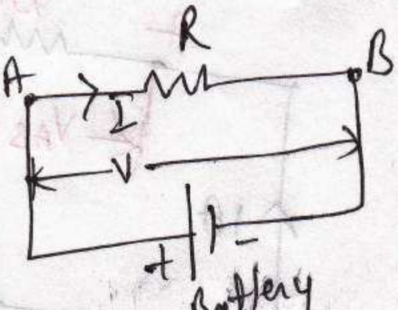
→ Thus the potential difference b/w points A & B is 7 Volts.

⊗ Ohm's Law:-

Defination: the current flowing between any two points of a conductor (or circuit) is directly proportional to the potential difference across them, provided physical conditions i.e temp. etc do not change.

Mathematically

$$I \propto V$$



fig(5)

$$\frac{V}{I} = \text{constant}$$

$$\frac{V}{I} = R \quad , V = IR \quad , I = \frac{V}{R}$$

V = IR (In a Ckt, when current flows through a Resistor, the p.d across the Resistor is known as voltage drop across it)

⊛ D.C Circuit :-

Defination → The closed path in which direct current flows is called d.c circuit.

→ A simple d.c circuit is shown in fig (6)

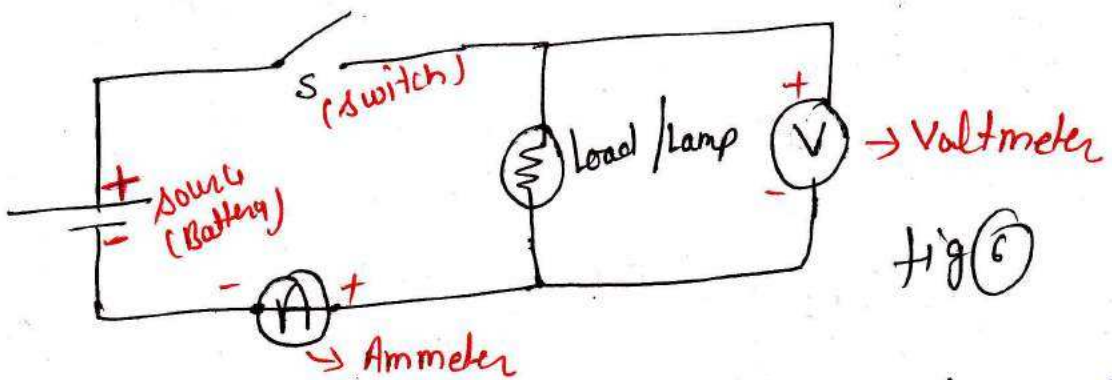


fig (6)

→ The load Resistors are connected in series, parallel or series - parallel combination as per the Requirements.

⊛ Series Circuit :-

The circuit in which number of Resistors are connected end to end so that same current flows through them is called series circuit.

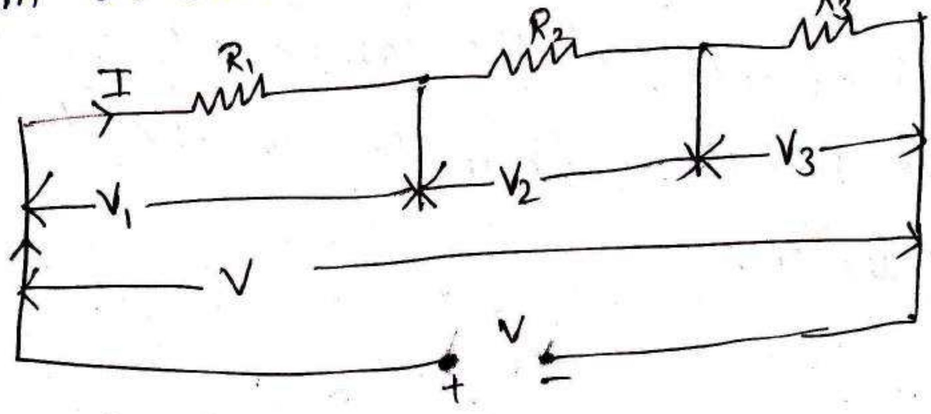


fig (7)

fig (7) shows a simple series circuit.

→ In the circuit, three Resistors R_1, R_2 & R_3 are connected in series across a supply voltage of v volts.

→ The same current I is ~~fort~~ flowing through all the three Resistors.

→ If V_1, V_2 & V_3 are the voltage drop across the three Resistors R_1, R_2 & R_3

$$V = V_1 + V_2 + V_3 \qquad (\because V = IR)$$

$$V = IR_1 + IR_2 + IR_3$$

$$IR = IR_1 + IR_2 + IR_3$$

$$\boxed{R = R_1 + R_2 + R_3}$$

total Resistance = sum of individual Resistance

Application:- → In the Marriages for decoration purposes where a number of low voltage lamps are connected in series.

→ In this circuit, all the lamps are controlled by a single switch, they can not be controlled individually.

⊛ Parallel Circuit:-

→ The circuit is which one end of all the Resistors is joined to a common point and the other ends are also joined to another common point, so that different current flows through them is called parallel circuit.

fig (8) shows a simple parallel circuit.

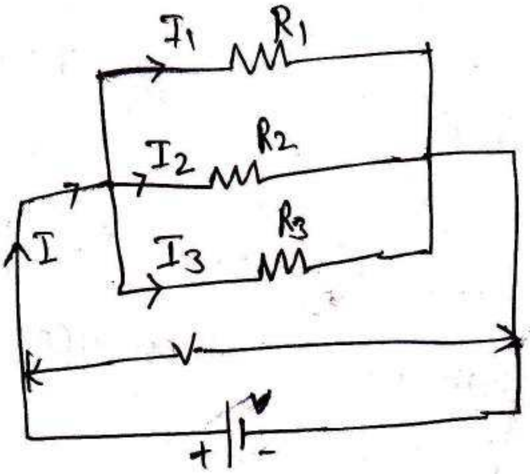


fig (8)

→ In this circuit, three Resistors R_1, R_2 & R_3 are connected in parallel across a supply voltage of V volts.

→ The current flowing through them is I_1, I_2 & I_3

→ Total current drawn by the circuit

$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$\left(\because V = IR, \; I = \frac{V}{R} \right)$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Series - Parallel Circuit :

→ The circuit in which series & parallel circuits are connected in series is called series - parallel circuit. fig (9) shows a simple series - parallel circuit.

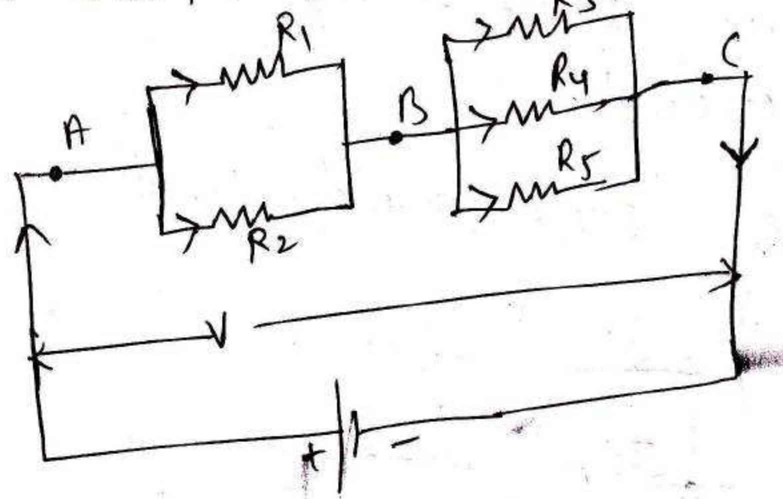


fig (9)

→ In this circuit two Resistors R_1 & R_2 are connected in parallel with each other across terminal AB.

→ The other three Resistors R_3, R_4 & R_5 are connected in parallel with each other across terminal BC.

→ the two Groups of Resistors R_{AB} & R_{BC} are connected in series with each other across the supply voltage of V volts.

$$\frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

or $$R_{AB} = \frac{R_1 R_2}{R_1 + R_2}$$

Similary $\frac{1}{R_{BC}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$

$$R_{BC} = \frac{R_3 R_4 R_5}{R_3 R_4 + R_4 R_5 + R_5 R_3}$$

∴ Total Resistance of the circuit

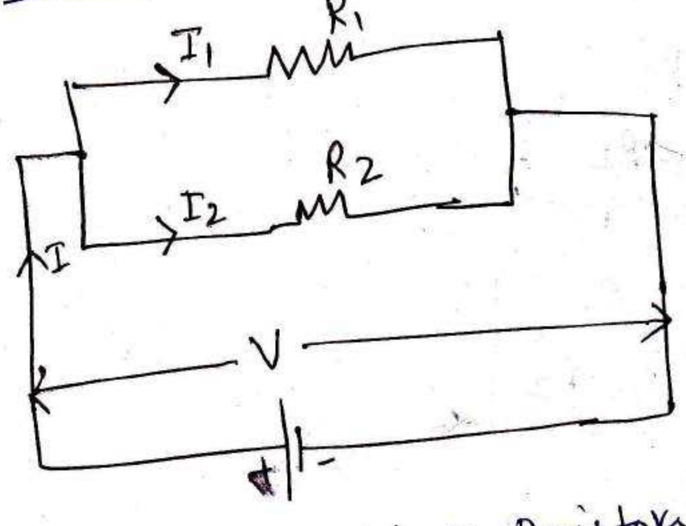
$$\boxed{R = R_{AB} + R_{BC}}$$

(*) Division of Current in parallel circuits:-

OR

Current Division Rule (CDR) :-

1) when two Resistance are Connected in parallel:-



Above fig shows two Resistors having Resistance R_1 & R_2 Connected in parallel across a supply voltage of V volts. Let the Current in each branch be I_1 & I_2.

→ According to ohm's Law

$$I_1 R_1 = I_2 R_2 = V$$

$$\begin{cases} V = I_1 R_1 \\ V = I_2 R_2 \end{cases}$$

$$\frac{I_1}{I_2} = \frac{R_1}{R_2}$$

$$IR = I_1 R_1 = I_2 R_2$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$ (total Resistance)

$$I_1 R_1 = IR$$

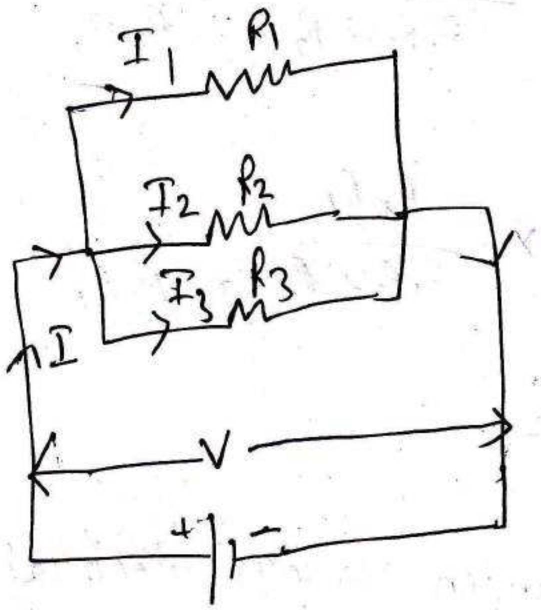
$$I_1 = I \times \frac{R_1 R_2}{(R_1 + R_2) \times R_1}$$

$$\boxed{I_1 = I \frac{R_2}{R_1 + R_2}}$$

Similary

$$\boxed{I_2 = I \frac{R_1}{R_1 + R_2}}$$

ii) When three Resistors are connected in parallel.



→ Above fig shows three resistors having Resistance R_1, R_2 & R_3 connected in parallel across a supply Voltage of v volts.

→ Let the current in each branch be I_1, I_2 & I_3

→ According to ohm's law.

$$I_1 R_1 = I_2 R_2 = I_3 R_3 = IR = V$$

R → total Resistance

$$R = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

$$I_1 R_1 = IR$$

$$\boxed{I_1 = I \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}}$$

Similary

$$I_2 = I \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

$$I_3 = I \times \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

⊛ Electrical Network:-

→ ~~The arrangement by which various electrical energy sources, like Resistances,~~

→ The arrangement by which various electrical element (Resistor, Inductor, capacitor, voltage source, current source) and other parameters are connected together is called electrical circuit or electrical Network.

→ We May classify circuit elements in two categories
- → Passive element
- → Active element

1) Active element:

The element which supplies energy to the circuit is called Active element.

eg:- Voltage source & current sources, generator etc.

In fig ⊗, E_1 & E_2 are the Active element.

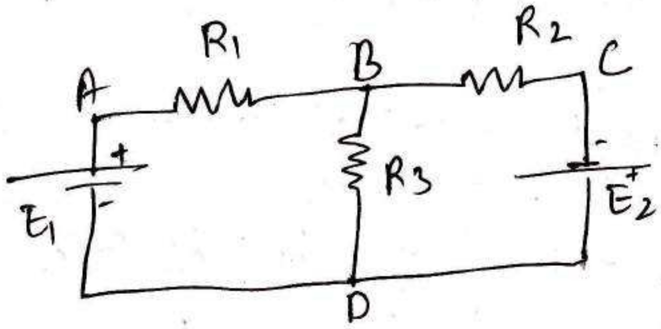


fig (*)

2) Passive element:-

The element which Receives energy is called Passive element. And then either convert in to heat (R) or stored it in an electric (C) or magnetic (L) field is called passive element.

eg ⇒ Resistor, capacitor & Inductor etc.

In fig (*) R_1, R_2 & R_3 are the passive elements of the Network.

→ These are also known as parameters of the Network.

3) Node:-

A node is a point in the Network where two or more circuit elements are joined.

In fig (*) A, B, C & D are the nodes.

4) Junction:-

A Junction is a point in the Network where three or more circuit elements are Joined. In fact, it is a point where current is divided.

In fig ⊗ B & D are the Junctions.

5) Branch:- The part of a Network which lies between two Junction points is called branch.
In fig ⊗ D A B , B C D & B D are three Branches.

6) Loop:- The closed path of a Network is called a loop.
In fig ⊗ A B D A , B C D B & A B C D A are the three loops

7) Mesh :- The Most elementary form of a loop which can-not be further divided is called a mesh. In fig ⊗ A B D A & B C D B are two Meshes but A B C D A is the loop.

8) Bilateral Element:-
Conduction of current in both direction is an element (eg → R, C & L) with same magnitude is termed as Bilateral element.

9) Unilateral Element:-
Conduction of current in one direction is termed as unilateral (eg → Diode, Transistor) element.

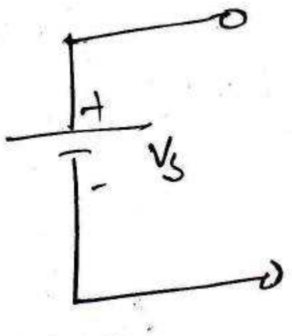
⊛ Electrical Energy Sources:-

→ There are two types of source of Electrical energy, the voltage source & the current source.

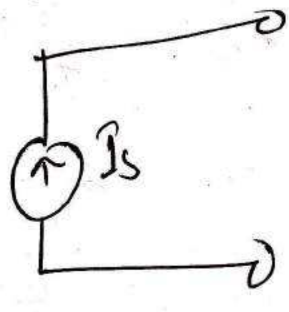
→ They are two-terminal element either independent or dependent.

Independent Sources:-

→ A source is said to be independent, when it does not depends on any other quantity in the circuit.



→ V_s → Independent voltage source



→ I_s → Independent Current source

$V_s = I_s =$ Constant Value

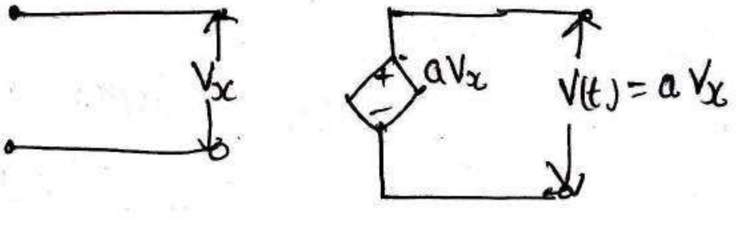
Dependent Sources:- (Controlled source):-

→ In this, source voltage or current depends on a voltage across or a current through some other element elsewhere in the circuit.

→ Sources, which exhibit this dependency, are called dependent sources.

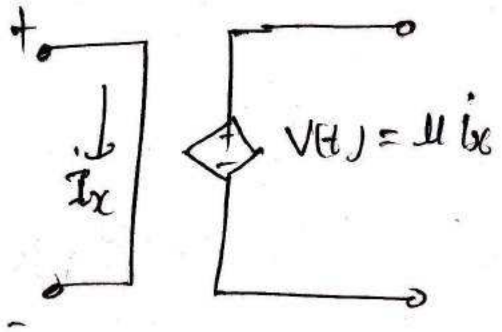
→ Both voltage and current types of sources may be dependent and either may be controlled by a voltage or a current.

→ Dependent source is Represented by a diamond () shaped symbol



↓

Voltage controlled voltage source



↓

Current controlled voltage source

Kirchhoff's Laws:-

Kirchhoff's first Law:- (KCL)

~~Defi~~ ~~So~~ this law relates the

→ ~~The~~ Current following through the circuit, it is also known as Kirchhoff's Current Law (KCL)

Defination:-

The algebric sum of all the currents meeting at a point or Junction is zero

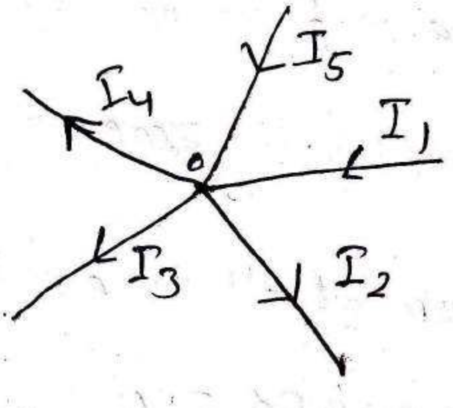


fig →

Mathematically:

$$\sum I = 0$$

→ As Algebric sum is to be taken, Consider

Incoming Currents → +ve

outgoing Currents → −ve

Apply KCL to the fig.

$$I_1 - I_2 - I_3 - I_4 + I_5 = 0$$

$$I_1 + I_5 = I_2 + I_3 + I_4$$

Incoming currents = outgoing Currents

Kirchhoff's Second Law:- (KVL)

Defination:-

In a closed circuit or mesh, the algebric sum of all the e.m.f plus the algebric sum of all the voltage drops is zero.

i.e Algebric sum of all the e.m.fs + Algebric sum of all the voltage drops = 0

Mathematically:-

$$\Sigma E + \Sigma V = 0$$

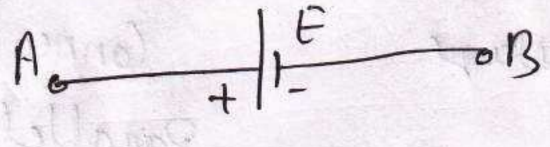
→ As Algebric sum of e.m.fs & voltage drops is to be taken, we have to determine their signs while tracing a circuit.

for this we always consider

A rise in potential as → +ve

A fall in potential as → −ve

(only for emf)



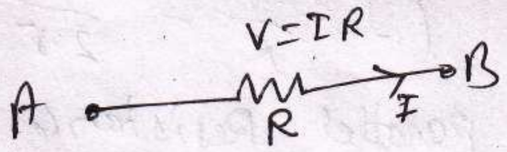
Tracing Branch A to B , E → −ve

" " B to A , E → +ve

Note :- (Direction of flow of current has no effect)

→ To determine sign for the voltage drop

$V = IR$



Tracing Branch from A to B , V → −ve

" " " B to A , V → +ve

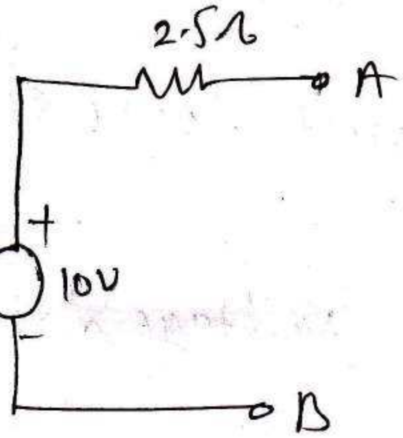
Note :- only the direction of flow of current determine the sign of V.

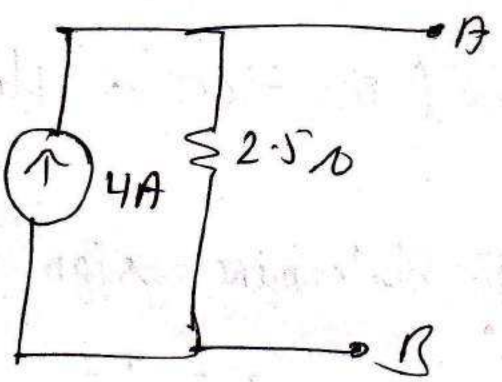
(*) Source Transformation :-

$V = IR$, $I = \frac{V}{R}$ →

↓ Resistance connected in series with voltage source

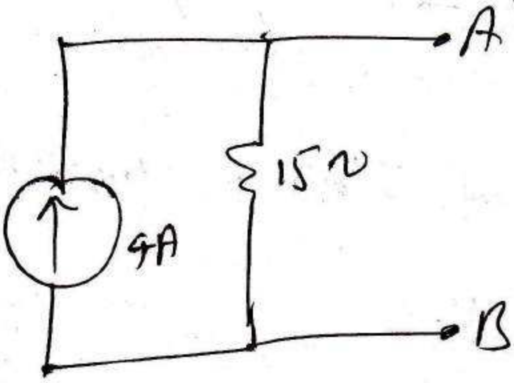
→ Resistance Connected in Parallel with current source.

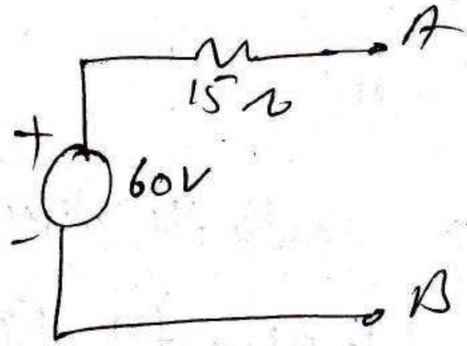




$\left(I = \frac{V}{R} = \frac{10}{2.5} = 4 \right)$

Q: Convert 4A source with its parallel Resistance of 15Ω in to its equivalent voltage source





↓ Current source

↓ Voltage source

$\left(V = IR \right.$
$V = 4 \times 15$
$= 60V$

* Maxwell's Mesh Current Method:-

→ In this method, mesh or loop currents are taken instead of Branch Currents

The following steps are taken while solving a Network by this Method.

1) The whole n/w is divided in to number of meshes. Each mesh is assigned a current having Continuous path. These mesh currents are drawn in clock-wise direction.

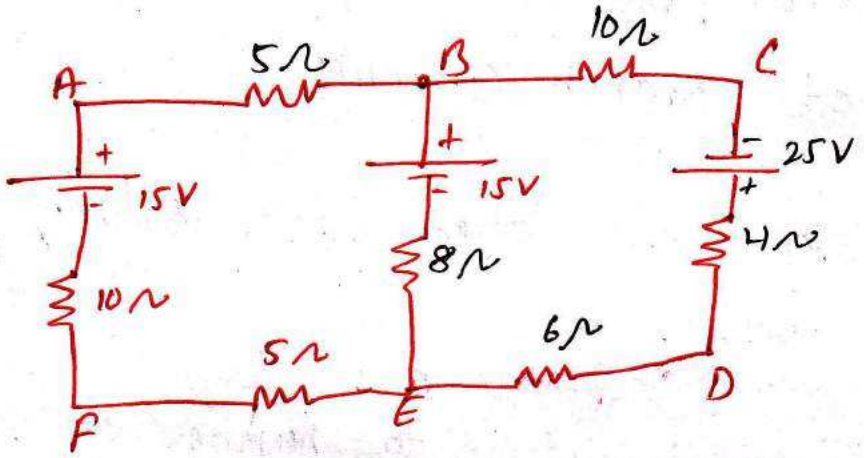
2) Write KVL eqⁿ for each mesh using the same signs as applied Kirchhoff's law

3) Number of equations must be equal to the number of unknown quantities.

4) Solve the equations & determine the mesh currents

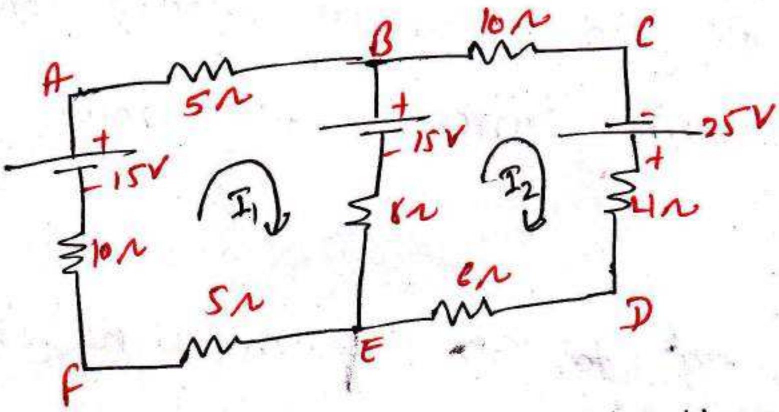
5) from mesh currents, determine the Branch currents.

Example:- Solve the Network shown in fig ① & find the current in each branch.



→ fig ①

Soln:-



→ fig ②

Let the current flowing through the two loops ABEFA & BCDEB be I_1 & I_2 as shown in fig ②

Apply KVL to different loops

Loop ABEFA

$$-5I_1 - 15 - 8(I_1 - I_2) - 5I_1 - 10I_1 + 15 = 0$$

$$\boxed{28I_1 - 8I_2 = 0}$$

or

$$\boxed{I_2 = 3.5I_1}$$

Loop BCDEB

$$-10I_2 + 25 - 4I_2 - 6I_2 - 8(I_2 - I_1) + 15 = 0$$

$$8I_1 - 28I_2 = 0$$

or $8I_1 - 28(3.5I_1) = -40$

$$I_1 = \frac{4}{9} = 0.444\ A$$

$$\boxed{I_1 = 0.444\ A}$$

$$I_2 = 3.5 \times I_1 = 3.5 \times \frac{4}{9} = 1.555\ A$$

$$\boxed{I_2 = 1.555\ A}$$

Current in Branch EFAB, $I_1 = .444\ A$

" " " BCDE, $I_2 = 1.555\ A$

" " " EB $= I_2 - I_1 = 1.11\ A$

Nodal Analysis:-

→ In this method, one of the nodes is taken as the Reference node & the other as independent node.

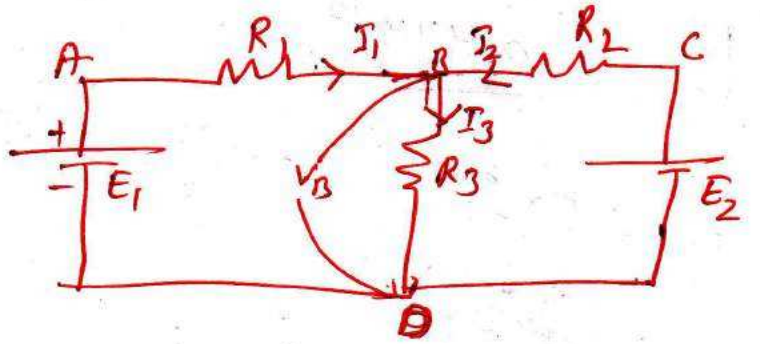
→ The voltage at the different independent nodes are assumed and the equations are written for each node as per KCL

→ After solving these equations, the node voltages are determined.

→ Then the Branch currents are determined.

→ Consider a ckt shown in fig①, where D & B are the two independent nodes.

→ Let D be the Reference node & the voltage of node B be V_B.



→ fig ①

→ According to KCL

$$I_1 + I_2 = I_3 \qquad \text{— eq}^n \text{ ①}$$

In Mesh AB DA, the p.d across R_1 is $E_1 - V_B$

p.d → potential difference.

$$\therefore \quad I_1 = \frac{E_1 - V_B}{R_1}$$

In Mesh BCDB, the p.d across $R_2 = E_2 - V_B$

$$\therefore \quad I_2 = \frac{E_2 - V_B}{R_2}$$

$$I_3 = \frac{V_B}{R_3}$$

Substituting these values in eqn (1), we get

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_2} = \frac{V_B}{R_3}$$

Rearranging the terms

$$V_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0$$

Since all other value are known except V_B, therefore calculate the value of V_B. And then determine the value of I_1, I_2, & I_3.

→ This method is faster as the Result are obtained by solving lesser number equations.

Example:- for the circuit shown in fig (1), find the current in various branches by nodal Analysis

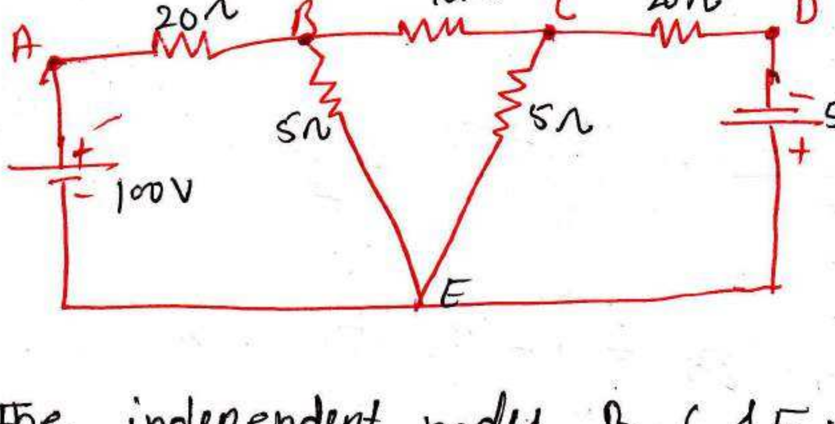


fig (1)

Soln:- The independent nodes B, C & E. Let E be the Reference node & V_B & V_C be the voltages at node B & C. Mark the current flowing through various branches as show in fig (2)

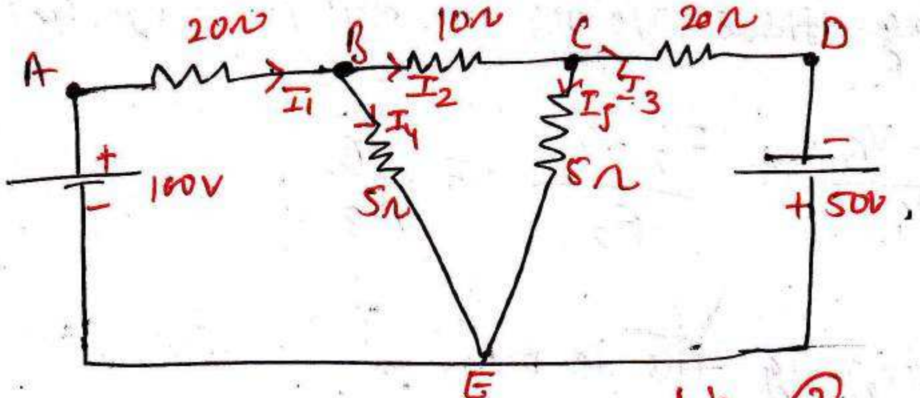


fig-③

At node B

$$I_1 = I_2 + I_4 \qquad \text{(using KCL)}$$

$$\frac{(100 - V_B)}{20} = \frac{V_B - V_C}{10} + \frac{V_B}{5}$$

$$V_B\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{5}\right) - \frac{100}{20} - \frac{V_C}{10} = 0$$

$$7V_B - 2V_C - 100 = 0 \qquad \text{— eq}^n \text{ ①}$$

At node C

$$I_2 = I_3 + I_5 \qquad \text{(using KCL)}$$

$$\frac{V_B - V_C}{10} = \frac{V_C + 50}{20} + \frac{V_C}{5}$$

$$\left(\because I_3 = V_C - V_D,\ V_D = -50,\ I_3 = V_C + 50\right)$$

$$V_C\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{5}\right) - \frac{V_B}{10} + \frac{50}{20} = 0$$

$$7V_C - 2V_B + 50 = 0 \qquad \text{— eq}^n \text{ ②}$$

Solving eq^n (i) & (ii), we get

$$V_B = \frac{40}{3} V = 13.33V$$

$$V_C = -\frac{10}{3} V = -3.33V$$

∴ Current in various Branches

$$I_1 = \frac{100 - V_B}{20} = 4.33A \quad \text{(from A to B)}$$

$$I_2 = \frac{V_B - V_C}{10} = 1.67A \quad \text{(from B to C)}$$

$$I_3 = \frac{V_C + 50}{20} = 2.33A \quad \text{(from C to D)}$$

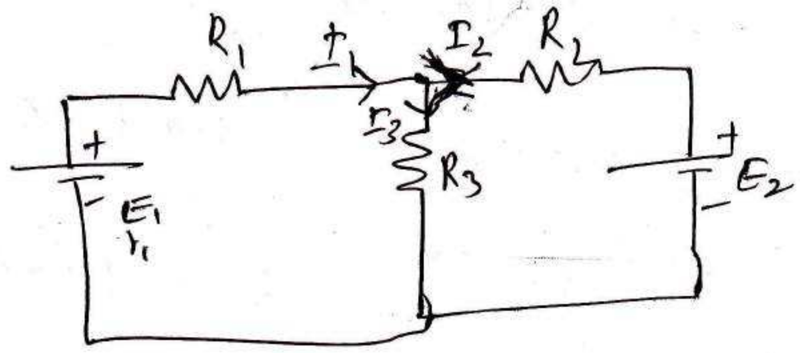
$$I_4 = \frac{V_B}{5} = 2.67A \quad \text{(from ~~D to E~~ E to B)}$$

$$I_5 = \frac{V_C}{5} = -0.67A \quad \text{(from E to C)}$$

Superposition theorem:-

→ According to this theorem, if there are two or (>/) more than two sources of e.m.fs acting simultaneously in linear Bilateral Network, the current flowing through any section is the algebraic sum of all the currents which should flow in that section if each source of e.m.f were considered separately and all other sources are replaced, for the time being, by their internal resistances.

→ for Instant, consider a circuit shown in fig (16) ; let the current flowing through various branches be as marked in fig (16).



→ fig (16)

According to superposition theorem

1) Consider only one source E_1 and Replace the other source E_2 by its internal Resistance. As its internal Resistance is not given, it is taken as zero (short-circuit).

2) Draw the circuit as shown in fig 16 (a) and determine currents in various section as I_1', I_2' & I_3' Respectively.

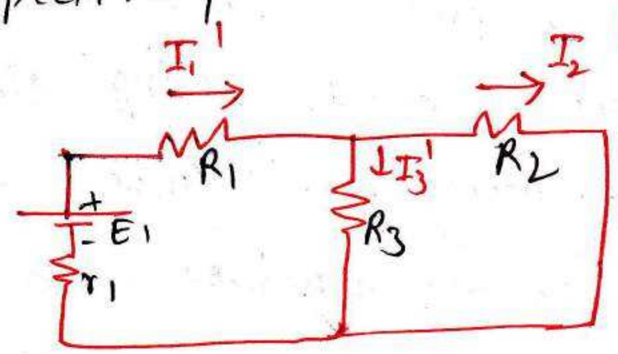
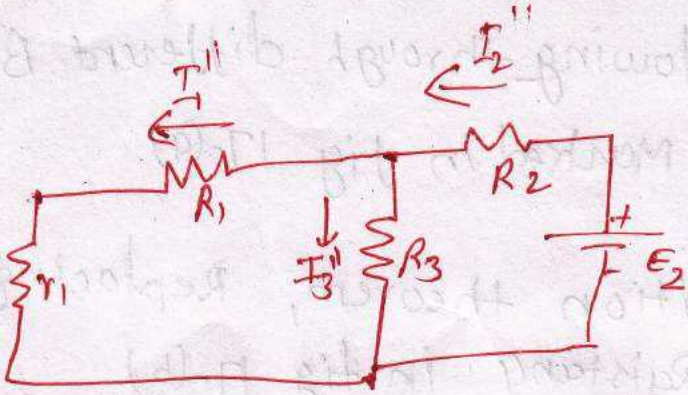


fig 16-(a)

3) Then consider the other source E_2 & Replace the source E_1 by its internal resistance r_1 as shown in fig 16-(b).



→ fig 16-(b)

4) Determine the currents in various section as I_1'', I_2'' & I_3''.

Actual flow of current in various sections.

$$I_1 = I_1' - I_1'' \,, \quad I_2 = I_2' - I_2''$$

$$I_3 = I_3' + I_3''$$

Example: Determine the Branch currents by superposition theorem in the Network shown in fig (17).

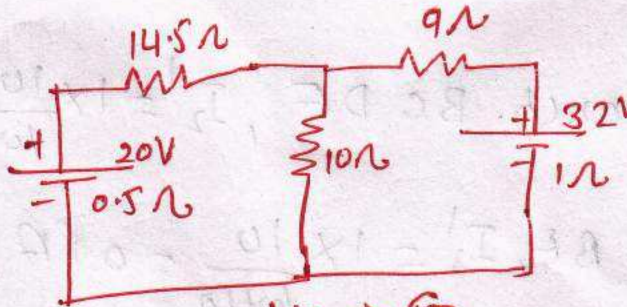
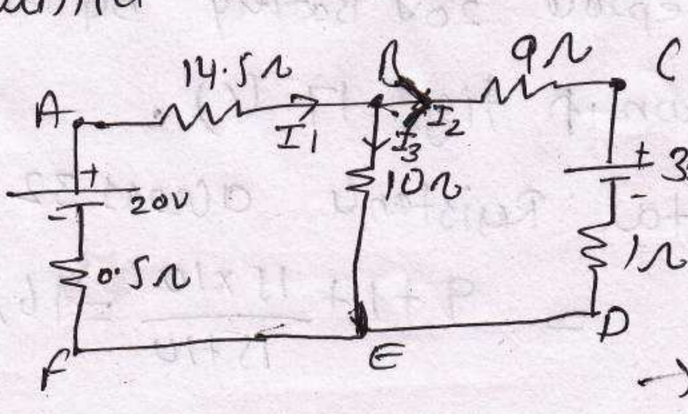


fig → (17)

Sol^n:- The simplified ckt is shown in fig 17-(a).



→ fig 17-(a)

→ Let the Current flowing through different Branches be I_1, I_2 & I_3 as Marked in fig 17-(a).

→ According to superposition theorem, Replacing 32 V Battery by its internal Resistance in fig 17-(b)

Total Resistance across 20V sources

$$= 14.5 + 0.5 + \frac{10 \times (9+1)}{10+(9+1)} = 20\Omega$$

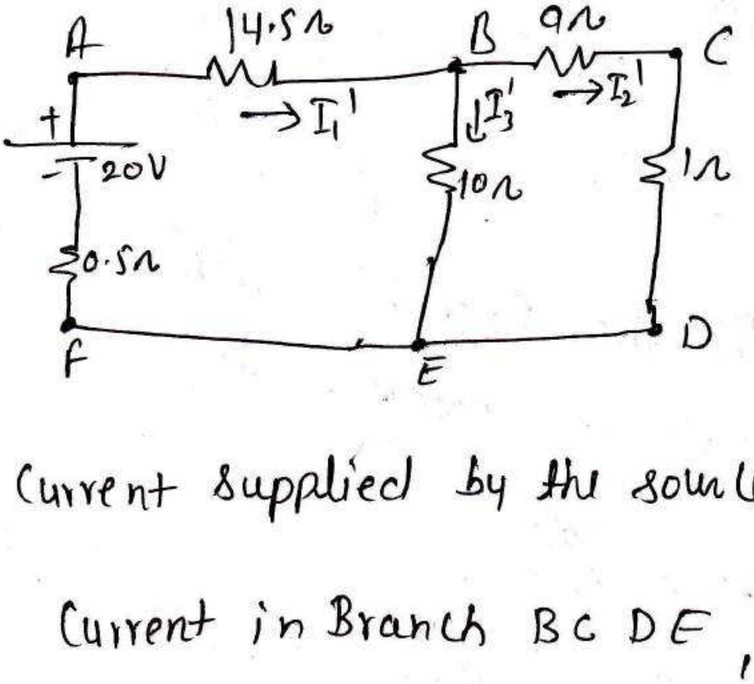


fig → 17-(b)

→ Current supplied by the source, $I_1' = \frac{20}{20} = 1A$

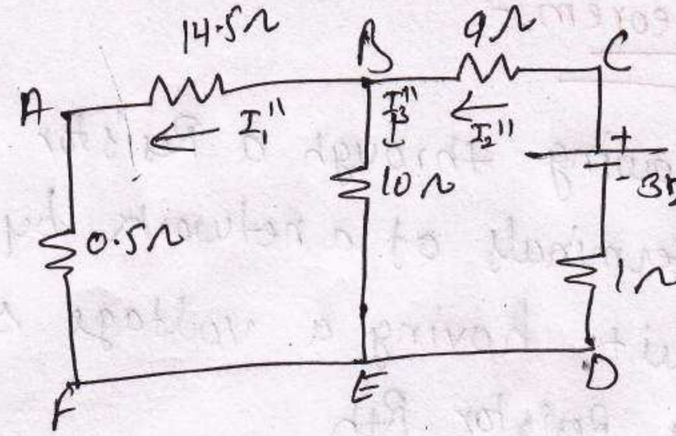
Current in Branch BCDE, $I_2' = 1 \times \frac{10}{10+10} = 0.5A$

Current in Branch BE, $I_3' = 1 \times \frac{10}{10+10} = 0.5A$

→ Now Replace 20V Battery by its internal Resistance shown in fig 17-(c).

total Resistance across 32 V, source

$$= 9 + 1 + \frac{15 \times 10}{15+10} = 16\Omega$$



→ fig 17-(c)

Current supplied by the source

$$I_2'' = \frac{32}{16} = 2A$$

Current in Branch BAFE, $I_1'' = 2 \times \frac{10}{10+15} = 0.8A$

Current in Branch BE, $I_3'' = 2 \times \frac{15}{10+15} = 1.2A$

The actual flow of current in various Branches is obtained by superimposition of the sets of currents.

$I_1 = I_1' - I_1'' = 1 - 0.8 = 0.2A$

$I_2 = I_2' - I_2'' = 0.5 - 2.0 = -1.5A$

$I_3 = I_3' + I_3'' = 0.5 + 1.2 = 1.7A$

(*) Thevenin's Theorem:-

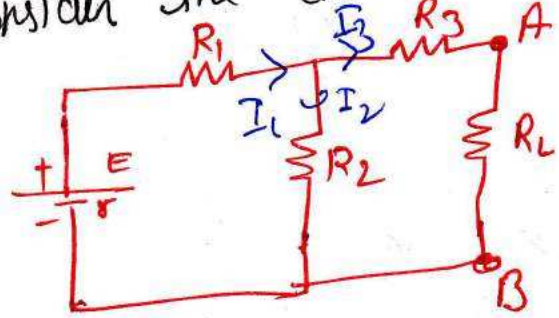
→ The current flowing through a Resistor connected across any two terminals of a network by an equivalent circuit having a voltage source E_{th} in series with a Resistor R_{th}

Where,

E_{th} → The open circuit voltage between the Required two terminals called Thevnin voltage,

R_{th} → The equivalent Resistance of the Network as seen from the two terminals with all other sources Replaced by their internal Resistances called Thevenin Resistance.

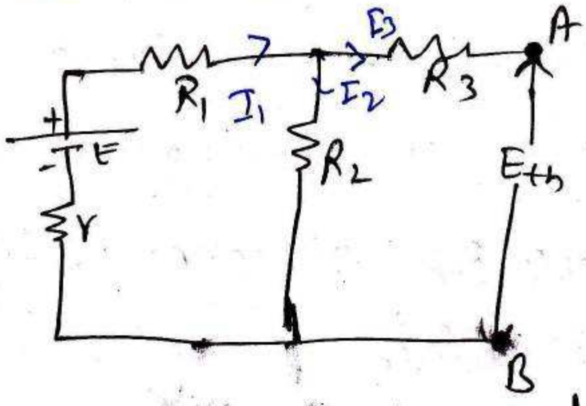
→ Consider the ckt shown in fig (18)



→ fig (18)

To determine the current through load Resistance R_L proceed with the following steps.

1) Remove the Resistance R_L in which current is to be determined, thus creating an open circuit between terminals A & B as shown in fig - 18-(a)



→ fig 18-(a)

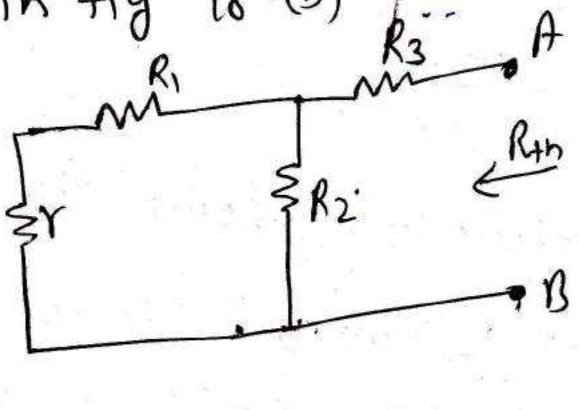
2) Determine the open circuit voltage (Thevenin Voltage Eth) between the terminals A & B

i.e

Voltage across $R_2 = I_2 R_2$

$$E_{th} = \left(\frac{E}{r+R_1+R_2}\right) R_2$$

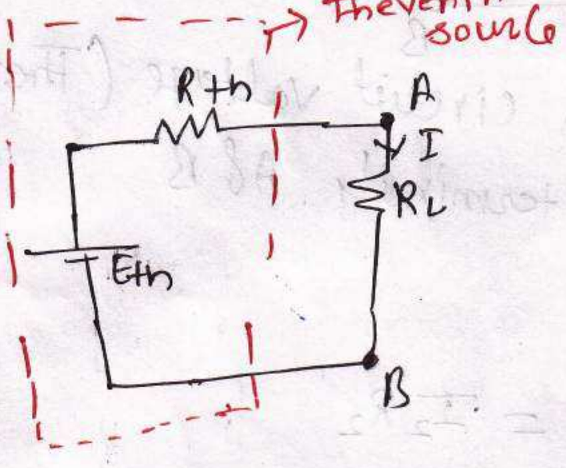
3) Replace the source (Battery) by its internal Resistance and determine the Resistance Rth (Thevenin Resistance) of the Network as seen from the terminals A & B as shown in fig 18-(b)



→ fig 18-(b)

$$R_{th} = \frac{(r+R_1)\cdot R_2}{(r+R_1)+R_2} + R_3$$

4) Replace the entire Network by a single Thevenin voltage source having an e.m.f E_{th} and internal Resistance R_{th} as shown in fig 18-(c)



→ fig 18-(c)

5) Connect the load Resistance R_L back to its terminals A & B from where it was removed

6) Determine the current flowing through the load Resistance R_L & by applying ohm's law

i.e $$I = \frac{E_{th}}{R_{th} + R_L}$$

Example:- for a Network shown in fig (19) Determine the current flowing through R_L when the value of load Resistance is i) 3Ω ii) 6Ω & iii) 9Ω

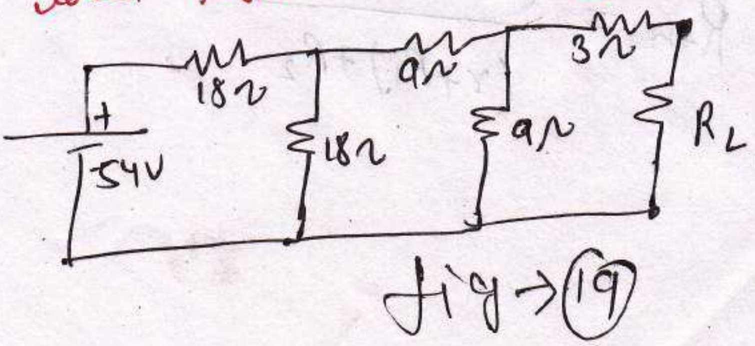
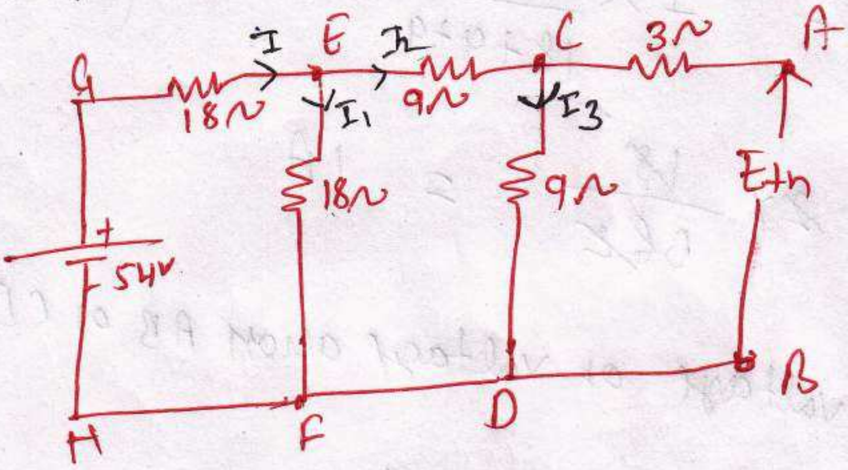


fig → (19)

Solution:- To determine voltage E_{th}, load Resistance R_L is Removed shown as fig 19-(a)



→ fig 19-(a)

Thevenin voltage across AB or CD

$$E_{th} = I_3 \times 9\Omega$$

Using Current Division Rule

$$I_3 = I \times \frac{18}{18+9+9}$$

$$I = \frac{V}{R}$$

$$R = 18 + \frac{(9+9)\times 18}{9+9+18} = 27\Omega$$ (total Resistance across 54 V Battery)

$$I = \frac{54}{27} = 2A$$

$$\boxed{I = 2A}$$ → Current supplied by the Battery

Current flowing through circuit E L D F or C D

$$I_3 = I \times \frac{18}{18+9+9} \quad (\because I = 2A)$$

$$= 2 \times \frac{18}{36} = 1A$$

∴ Thevenin voltage or voltage across AB or CD

$$E_{th} = 1 \times 9 = 9V$$

To determine Thevenin Resistance R_{th}, the source (Battery) is Replaced by its internal Resistance (zero in this case) as shown in fig 19-(b)

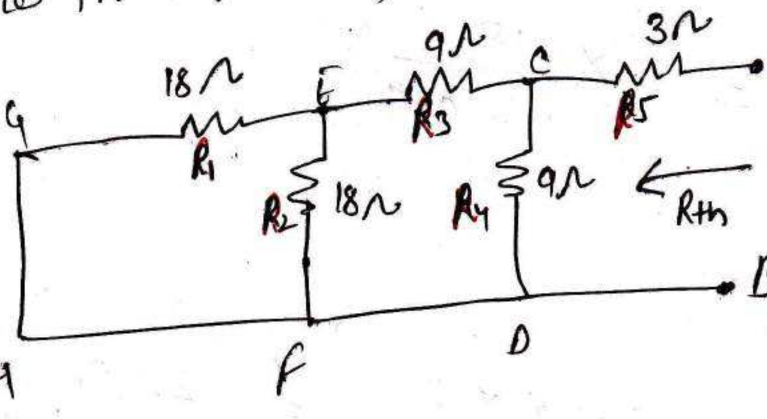


fig 19-(b)

Looking towards terminal A & B, Thevenin Resistance

$$R_{th} = \frac{\left(\frac{18 \times 18}{18+18} + 9\right) \times 9}{\left(\frac{18 \times 18}{18+18} + 9\right) \times 9} + 3$$

~~∵ GE is in parallel with EF & ...~~

Note

R_1 is in parallel with R_2

so that $\frac{R_1 \times R_2}{R_1 + R_2}$, Now this is in series with R_3 so that $\frac{R_1 \times R_2}{R_1 + R_2} + R_3$, Now this is in parallel with R_4, so that

$$\frac{\left(\frac{R_1 \times R_2}{R_1 + R_2} + R_3\right) \times R_4}{\frac{R_1 \times R_2}{R_1 + R_2} + R_3 + R_4}$$

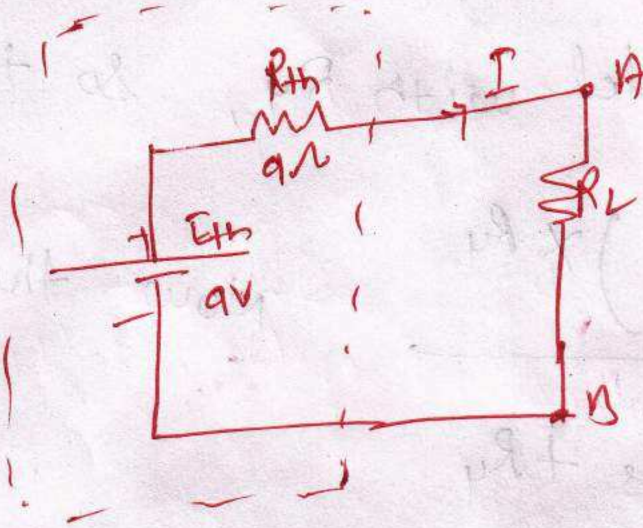
Now this is in series with R_5, so that

$$\left(\frac{\left(\frac{R_1 \times R_2}{R_1 + R_2} + R_3\right) \times R_4}{\frac{R_1 \times R_2}{R_1 + R_2} + R_3 + R_4} + R_5\right)$$

$$R_{th} = \frac{(9+9) \times 9}{9+9+9} + 3 = \frac{18 \times 9}{27} + 3$$

$$\boxed{R_{th} = 9\Omega}$$

The circuit is Reduced to a thevenin source of e.m.f $E_{th} = 9V$ & internal Resistance $R_{th} = 9\Omega$ as shown in fig 19-(c)



Current flowing through load Resistance

$$R_L, \quad I = \frac{E_{th}}{R_{th} + R_L}$$

i) when $R_L = 3\Omega$, $I_1 = 9/(9+3) = .75A$

ii) when $R_L = 6\Omega$, $I_2 = \frac{9}{9+6} = 0.6A$

iii) when $R_L = 9\Omega$, $I_3 = 9/(9+9) = 0.5A$

* Norton's Theorem:-

→ The current flowing through a resistance connected across any two terminals of a Network can be determined by Replacing the whole Network by an equivalent circuit of a current source having a current o/p of I_N in parallel with a Resistance R_N

I_N = The short - circuit current supplied by the source that would flow b/w the two selected terminals when they are short - circuited. It is generally called Norton's current

R_N = The equivalent Resistance of the Network as seen from the two terminals with all other e.m.f sources replaced by their internal Resistances and current sources replaced by open circuit. It is Generally called Norton's Resistance.

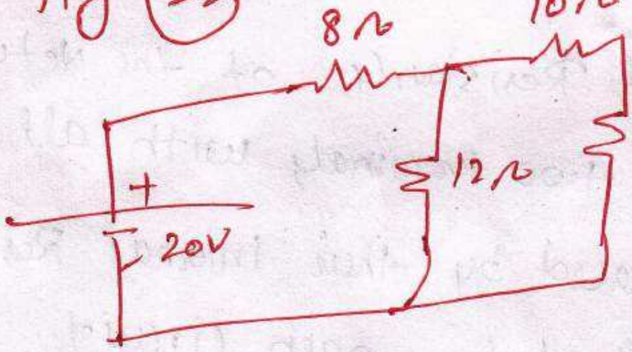
Steps to Determine Norton's equivalent circuit :-

1) Short circuit the terminals across which the load Resistor is connected & calculate the current which would flow between them. This is the current I_N.

2) Redraw the Network Replacing each voltage ~~both~~ sources By a short circuit in series with its internal Resistance if any and each current source by an open circuit in parallel with its internal Resistance

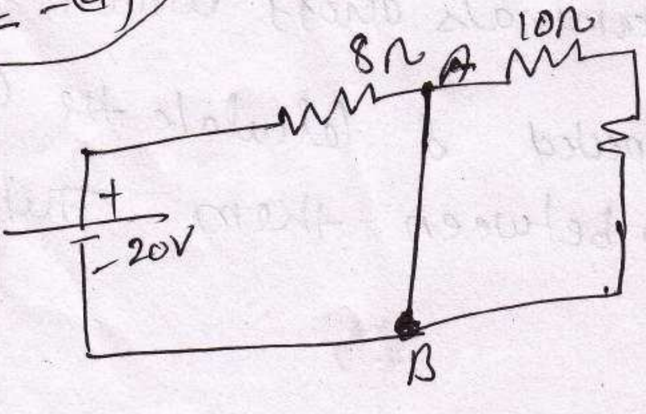
3) Determine the Resistance R_N of the Network as seen from the Network terminals. (Its value is the same as that of R_{th}).

<u>Example:</u> using Norton's theorem determine the Current in 12Ω Resistor in the Network shown in fig (22)



→ fig (22)

<u>Sol^n:</u> The 12Ω Resistor is Removed & terminals AB are short-circuited as shown in fig (22-(a))

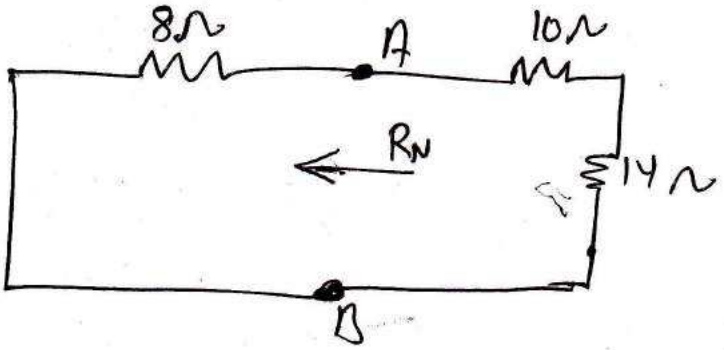


→ fig [22-(a)]

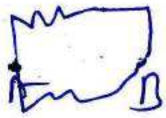
Since terminals AB are short circuit I_N is limited by only 8Ω Resistor.

$$I_N = \frac{20}{8} = 2.5A$$

→ The voltage source is Replaced by its ~~terminals~~ internal Resistance (zero in this case) and the Network is redrawn as shown in fig 22-(b) keeping terminal A - B open circuited



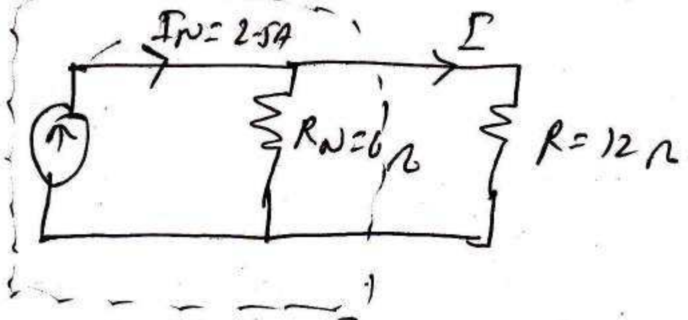
→ fig 22-(b)



Resistance of the Network as seen from terminal A & B

$$R_N = \frac{8 \times (10+14)}{8+(10+14)} = 6Ω$$

The Norton's current source is shown in fig 22-(c).



The current flowing through 12Ω Resistor

$$I = I_N \times \frac{R_N}{R_N + R_L}$$

$$= 2.5 \times \frac{6}{6+12}$$

$$I = 0.833A$$

Reciprocity theorem:-

This theorem allows the transfer of a source from one branch in the circuit to another branch.

⇒ It may be stated as in any linear bilateral n/w

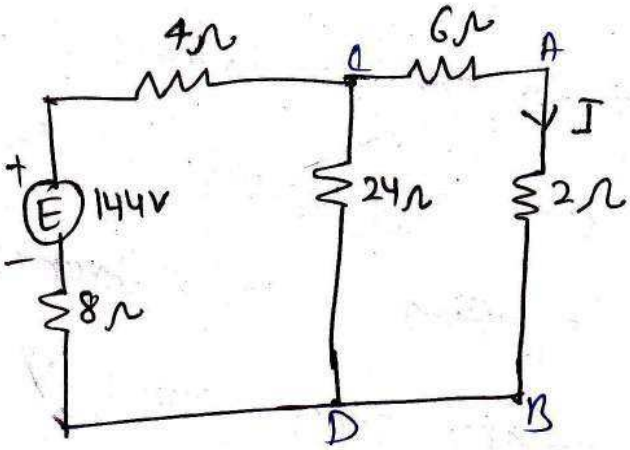
→ If a source of emf E acting in any branch X produces a current I in any other branch Y, then the same e.m.f E when shifted to branch Y would produce the same current I in the first branch X.

→ In fact this theorem states that E & I are interchangeable. The ratio of E/I is constant and is known as transfer Resistance.

~~Note:- while shifting the source of em.f E from one branch to the other, its internal Resistance is not shifted, internal Resistance of the source has to be kept in its original branch.~~

Note:- While shifting the source of emf E from one branch to other, its internal Resistance is not shifted. Internal Resistance of the source has to be kept in its original branch.

Question Verify the Reciprocity theorem for the Network shown in fig ①



→ fig ①

Soln

Refer fig ①

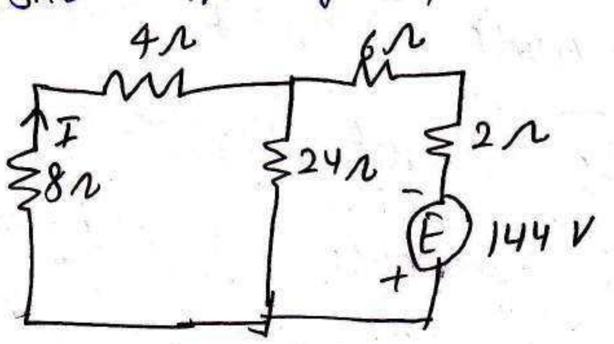
$$\text{Total circuit Resistance} = 8 + 4 + \frac{24(6+2)}{24(6+2)} = 18\Omega$$

$$\text{Current supplied by the source} = \frac{144}{18} = 8A$$

$$\text{Current in } 2\Omega \text{ Resistor} = 8 \times \frac{24}{24+8} \quad (\text{using CDR})$$

$$\boxed{I = 6A}$$

When source of emf is transferred to branch AB as shown in fig ②, then



→ fig ②

total Resistance $= \frac{(8+4)\times 24}{(8+4)+24} + 6+2$

$= 16\Omega$

Current supplied by the source $= \frac{144}{16} = 9A$

Current in 8-Ω Resistor, $I_1 = 9 \times \frac{24}{36}$

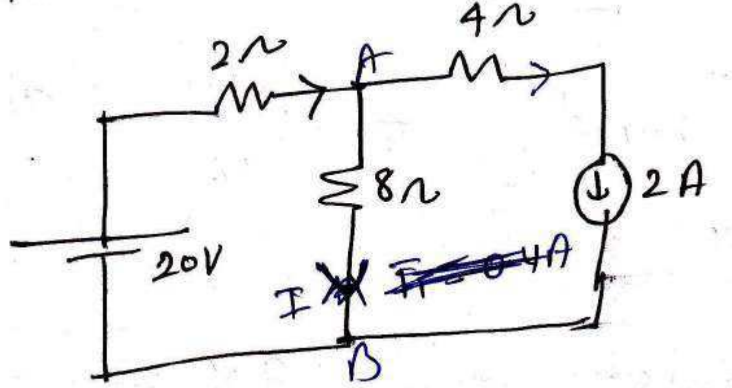
(using CDR)

$I = 6A$

This shows that E & I are interchangeable that verifies the Reciprocity theorem

transfer Resistance $= \frac{E}{I} = \frac{144}{6} = 24\Omega$

Ques: Determine the current 8-Ω Resistor in the following Network (see fig ③) using Superposition theorem.



→ fig ③

Solⁿ Replacing voltage source by its internal Resistance (i.e short circuit), a simplified ckt shown in fig ③-ⓐ

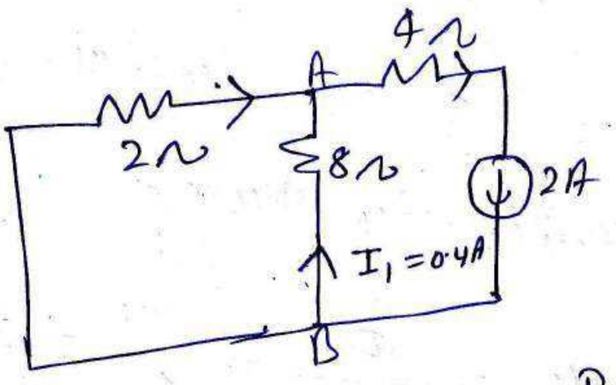


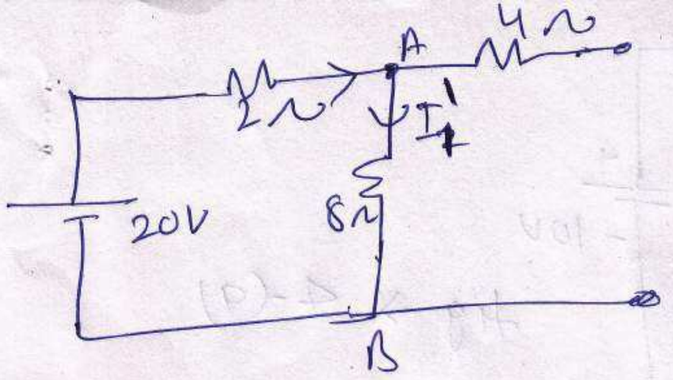
fig 3-(a)

Using current divider Rule

Current throug 8-Ω Resistor, $I_1 = 2 \times \frac{2}{2+8}$

$$I_1 = 0.4A$$

Again Replacing the current source by open circuit and Reducing the ckt as shown in fig 3-(b)



Current through 8Ω Resistor, $I_1' = \frac{20}{8+2}$

$I_1' = 2A$

Resultant current in 8Ω Resistor

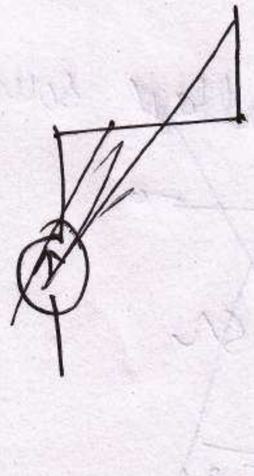
$I = I_1 + I_1'$

$I = (-0.4) + 2$

$I = 1.6A$

Question Using superposition theorem, Determine currents in all the Resistances of the N/w shown in fig

(4)



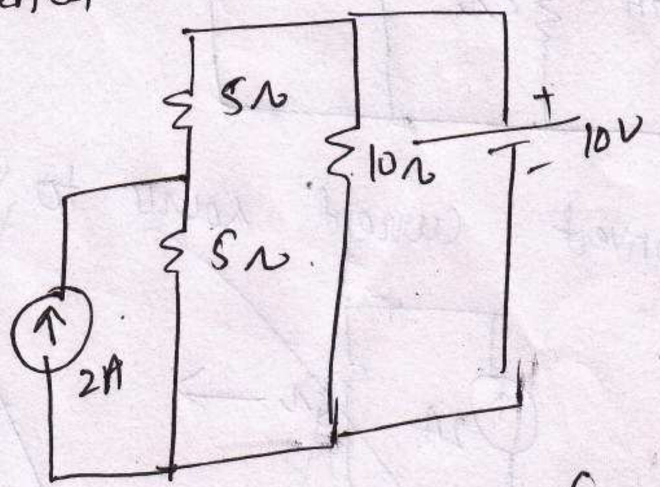


fig → (4)

Sol^n

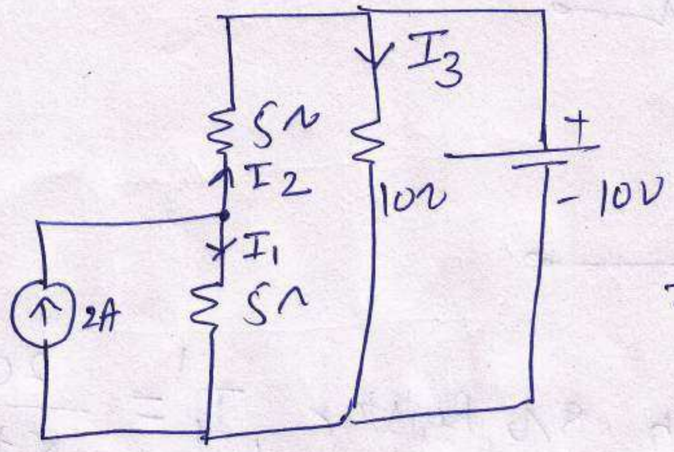
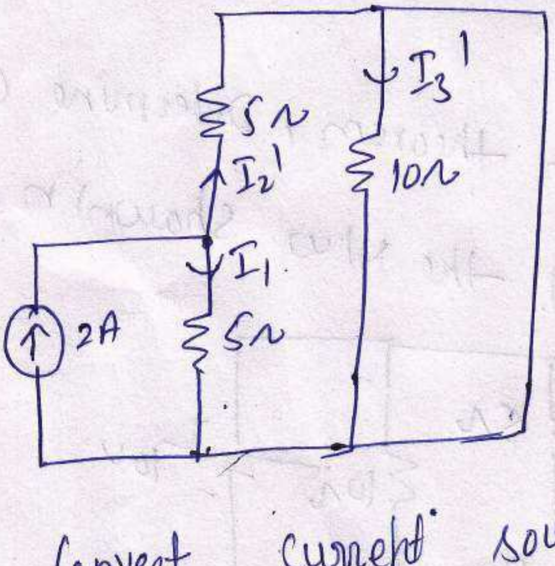


fig → 4-(a)

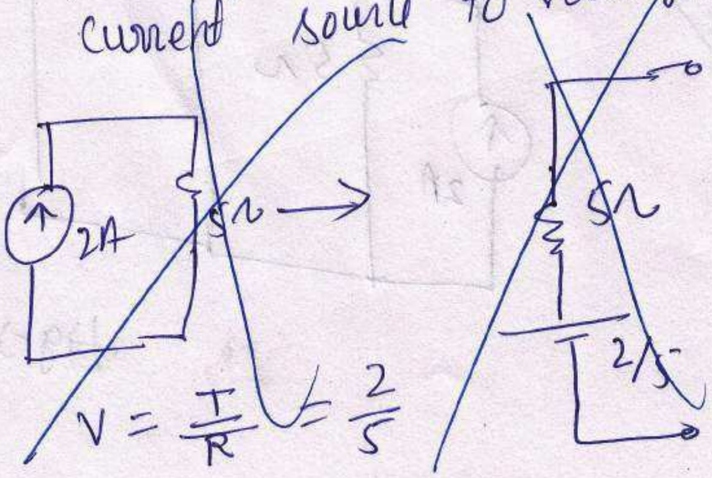
Let the current through various branches be I_1, I_2 & I_3 as shown in fig 4-(a)

Step-1 Consider 2A current source & short ckt the voltage source of 10V as shown in fig 4-(b)



→ fig 4-(b)

Convert current source to voltage source



Since 10-Ω Resistor is directly short circuited

$I_3' = 0$. further, by CDR

$I_1' = I_2' = 1A$ (both the 5-Ω Resistors are parallel with each-other)

Step-2 Considering only 10V voltage source & open circuiting the current source of 2A as shown in fig 4-(c)

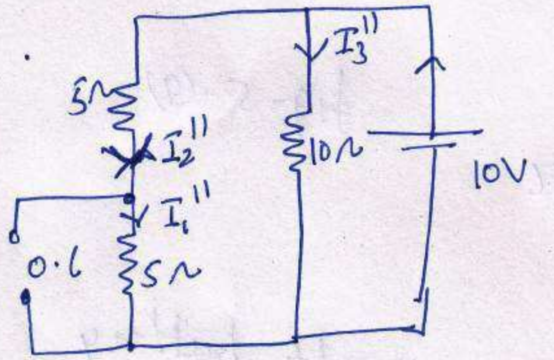


fig 4-(c)

$$I_3'' = \frac{10}{10} = 1A$$

$$I_1'' = I_2''$$

$$I_1'' = 1 \times \frac{10}{10} = 1A$$

$$I_2'' = 1A$$

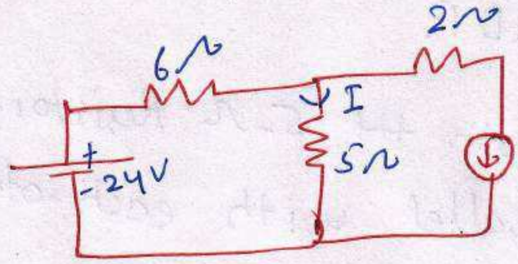
By superposition theorem

$$I_1 = I_1' + I_1'' = 1+1 = 2A$$

$$I_2 = I_2' + I_2'' = 1-1 = 0A$$

$$I_3 = I_3' + I_3'' = 0+1 = 1A.$$

Que: Find the current in the circuit given in figure (5)



→ fig (5)

Sol^n By applying superposition theorem, we get

Case I Considering 24V source only and denoting 4A source by its internal Resistance, that is open circuit, as shown in fig (5)-(a)

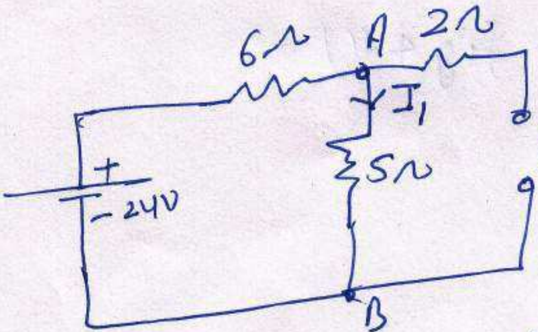


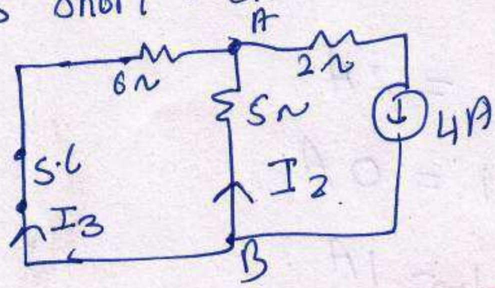
fig-5-(a)

Current supplied by 24 volt battery

$$I_1 = \frac{24}{6+5} = 2.18\,A \quad \text{(from A to B)}$$

Case-II Considering 4A current source only & Replacing 24V voltage battery by its internal Resistance, that is short-circuit as shown in fig 5-(b)

fig-5-b



Current divided in two paths: current in 5Ω Resistor

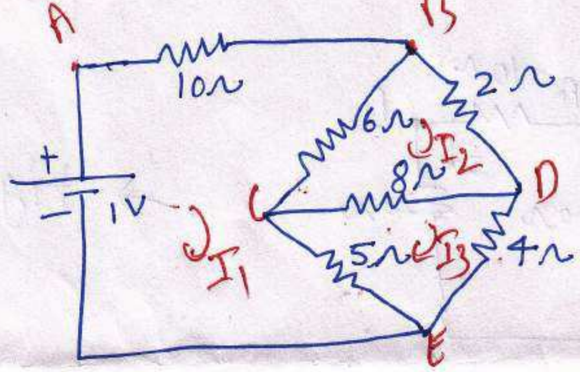
$$I_2 = \frac{6}{5+6} \times 4 = 2.18\,A \quad \text{(from B to A)}$$

$$\therefore \boxed{I = 2.18 - 2.18 = 0\,A}$$

Numericals (E.T)

(Ist unit)

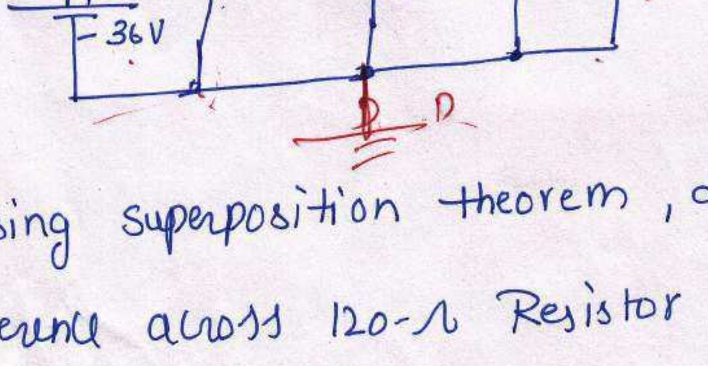
(1) Determine current supplied by the source in the Network shown in fig (1) using Maxwell's Mesh Current Method. ~~(40A)~~ (72mA)



→ fig (1)

(2) Using Method of Nodal Analysis, determine the current in various Resistors of the Network shown in fig (2)

(18A, 8A, 2A, 9A, 9A)



→ fig (2)

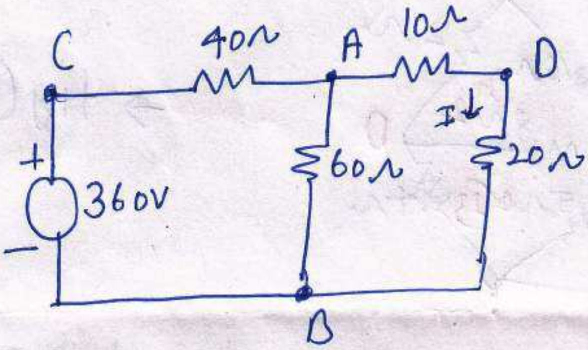
(3) Using superposition theorem, determine potential difference across 120-Ω Resistor in the network shown in fig (3)

(87.3V)

→ fig (3)

(4) Find the current in 20Ω Resistor in the Network shown in fig (4). If 360 V Generator is removed from the branch ACB & a Battery of 45 V is introduced in Branch BDA Determine the current in 40-Ω Resistor using Reciprocity theorem.

(0.5 A)

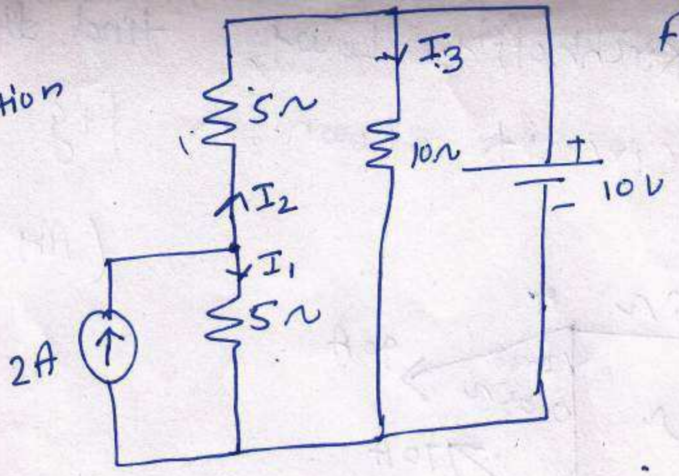


→ fig (4)

Ist Class Test

HMRITM, Hamidpur, New Delhi

Q:-① Superposition



Find the current in all Resistive Branches

(29)

$I_3' = 0$

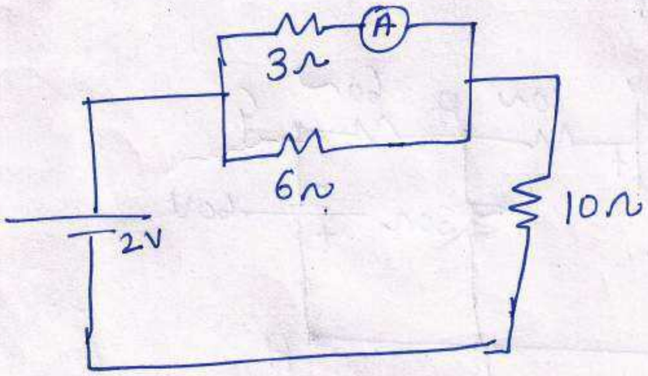
$I_1' = I_2' = 1A$

$I_3'' = 1A$, $I_1'' = 1A$

$I_1'' = I_2'' = 1A$

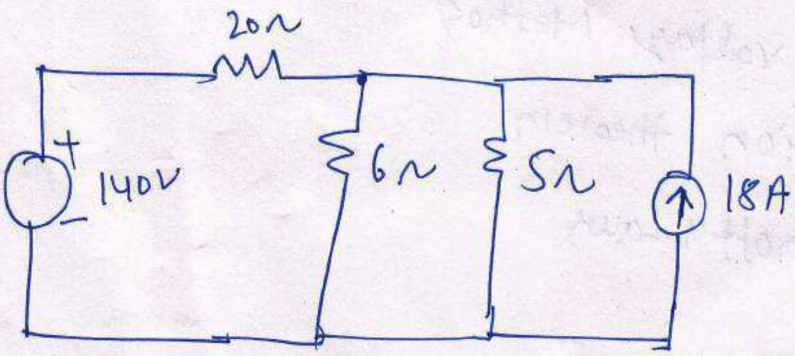
$I_1 = 2A$, $I_2 = 0A$, $I_3 = 1A$

Q:-2



(Reciprocity)

Q:- In the circuit shown in fig ① determine the current supplied by the DC Generator of 140V.

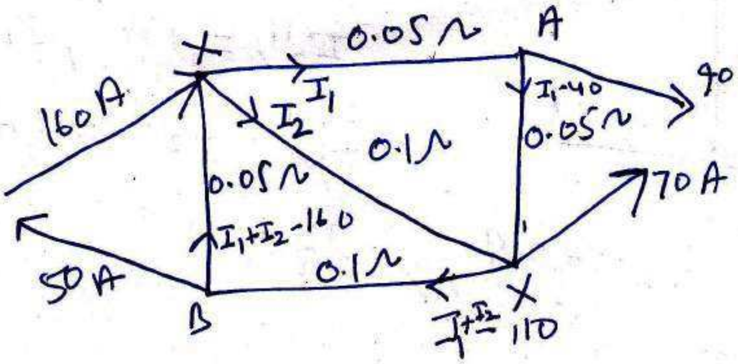


→ fig ①

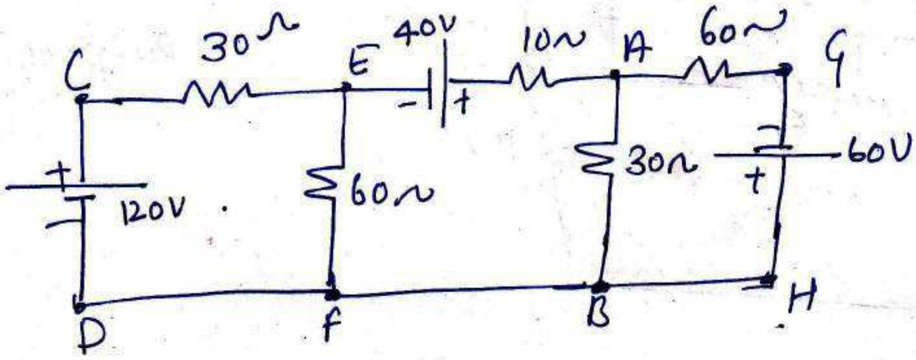
Ans → 4A

(x) By using Kirchhoff's Laws, find the current in XY in the circuit shown in fig

(Ans = 40A)



(x)



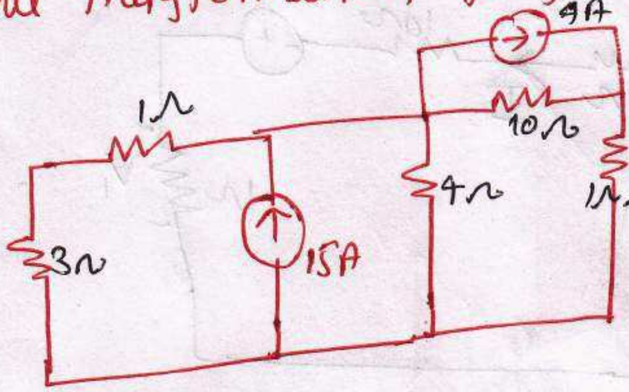
Find the the current in 30-Ω Resistor connected across AB in the circuit shown in fig using

(Ans = 1.2A)

(1) Maxwell's Mesh Current Method
(2) Nodal voltage Method
(3) Superposition theorem
(4) Kirchhoff's Laws

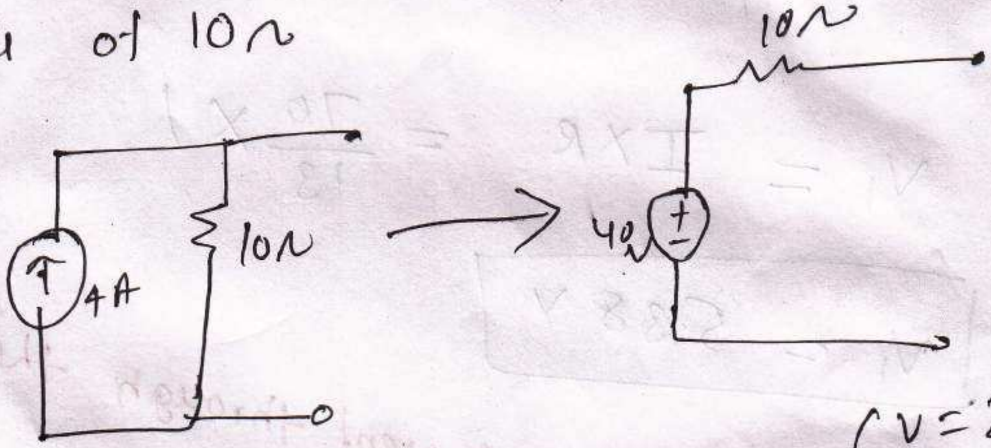
Numericals:-

Use the source transformation to find V_1 of fig (1)



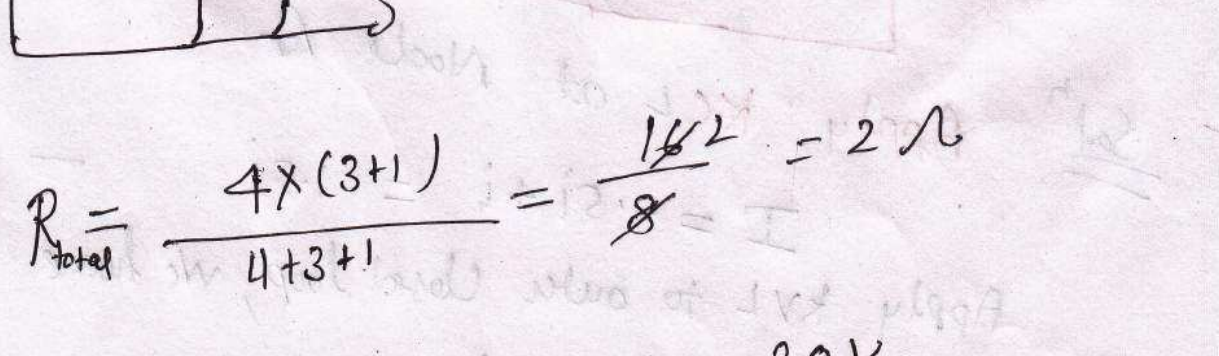
→ fig (1)

Sol^n (1) 4A current source in parallel with a Resistance of 10Ω



$(V = IR)$

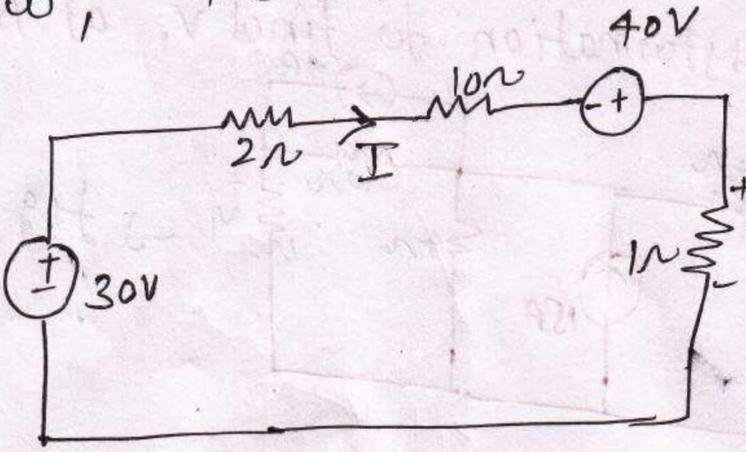
(2) 15A current source in parallel with a parallel combination of Resistance 4Ω & (3+1)Ω



$$\therefore R_{total} = \frac{4 \times (3+1)}{4+3+1} = \frac{16^{2}}{8} = 2\Omega$$

$$V = IR = 15 \times 2 = 30V$$

Now, the ckt is

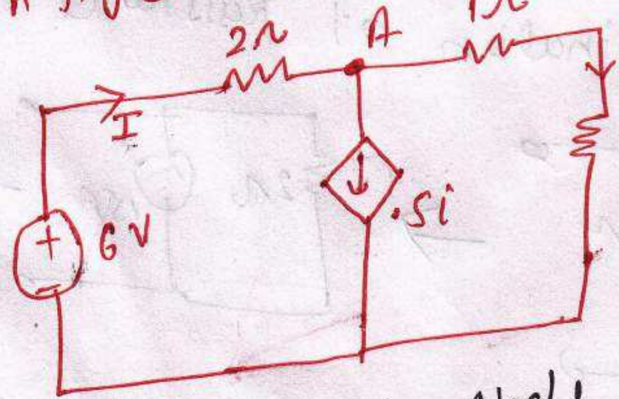


$$I = \frac{V}{R} = \frac{30+40}{2+10+1} = \frac{70}{13}$$

$$V_1 = I \times R = \frac{70}{13} \times 1$$

$$\boxed{V_1 = 5.38\ V}$$

Ques: ⑨ Determine the current through the 5Ω Resistor in fig ③



Sol^n Apply KCL at Node A

$$I = .5i + i = 1.5i \quad \text{— eq}^n \text{ ①}$$

Apply KVL to outer closed loop, we have

$$-2I - 1 \times i - 5i + 6 = 0$$

$$2I + 6i = 6$$

using eqn ①

$$2 \times 1.5 i + 6 i = 6 \qquad (\because I = 1.5 i)$$

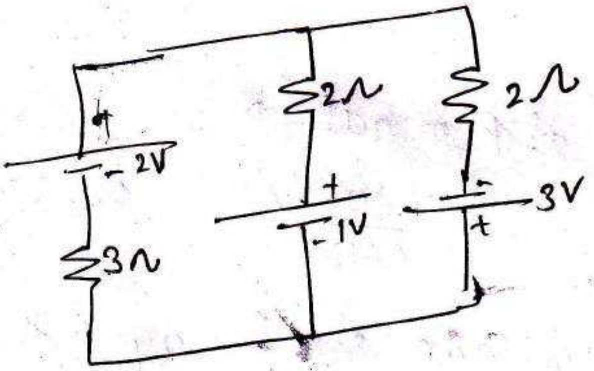
$$3i + 6i = 6$$

$$9i = 6$$

$$i = \frac{6}{9} = \frac{2}{3}$$

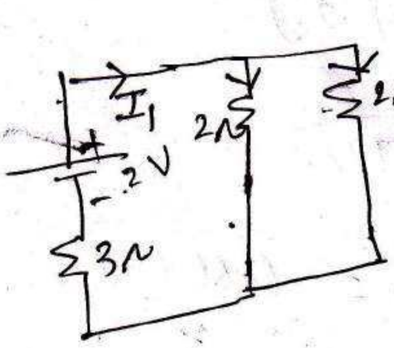
$$\boxed{i = 0.6 A}$$ Ans

⊗ Find the current through 3Ω Resistor using superposition theorem for the circuit as shown in fig



Soln Taking $E_2 = 0$, $E_3 = 0$

①

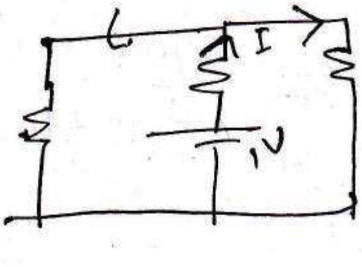


→ total current

$$I_1 = \frac{2}{3 + 2 \| 2} = \frac{2}{3 + \frac{2 \times 2}{2+2}}$$

$$I_1 = \frac{2}{4} = 0.5 A$$

②



Q:- Using Thevenin's theorem, find current I in the ckt show in fig (*)

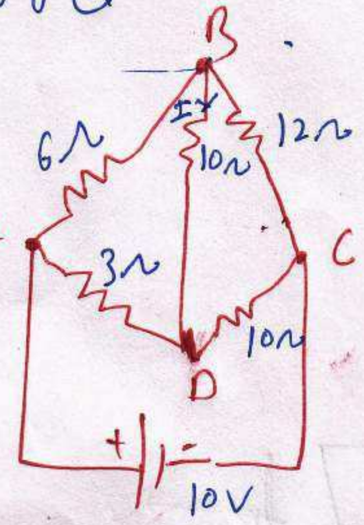


fig (*)

Sol^n To find V_{th} across BD, Remove the Resistance connected across BD as shown in fig (*)-(a)

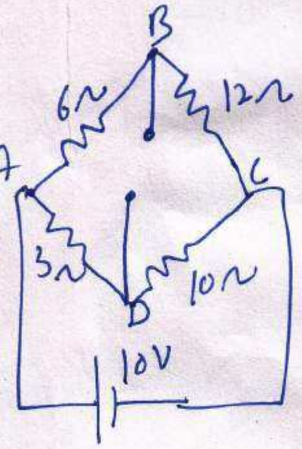


fig (*)(a)

∴ potential at terminal B is

$$V_B = 10 - \frac{10}{6+12} \times 6 = 6.667V$$

potential at terminal D,

$$V_D = 10 - \frac{10}{6+12} \times 3 = 8.333V$$

To determine R_{th}, Replace voltage source by its internal Resistance (i.e S.C) shown in fig (*)(b)

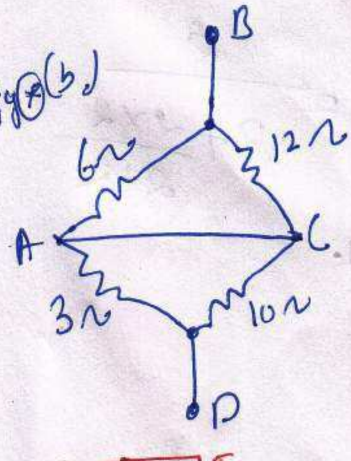
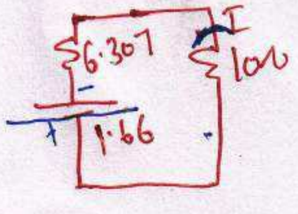


fig (*)(b)

$$R_{th} = (6 \| 12) + (3 \| 10) = \frac{6 \times 12}{6+12} + \frac{3 \times 10}{3+10}$$

$$R_{th} = \frac{82}{13} = 6.3077Ω$$

$$V_{BD} = V_{th} = V_B - V_D = 6.667 - 8.33 = -1.66$$



$$\Rightarrow \quad I = \frac{V_{th}}{R_{th} + 10} = \frac{-1.66}{6.3077 + 10}$$

$$I = -0.102$$

Numericals (Thevenin's & Norton's Theorem)

(1) Determine current through 6-Ω Resistance connected across A - B terminals in the electric circuit shown in fig (1) using Thevenin's theorem

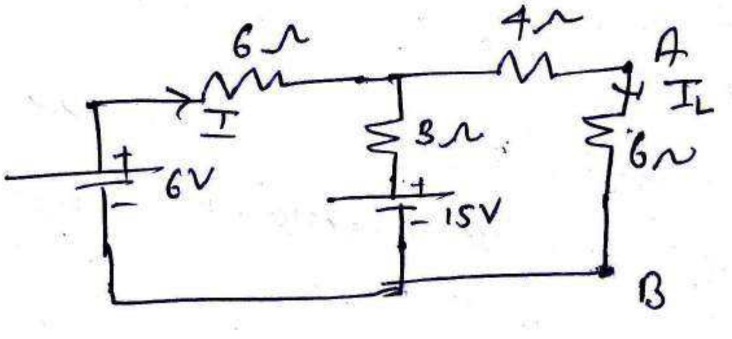
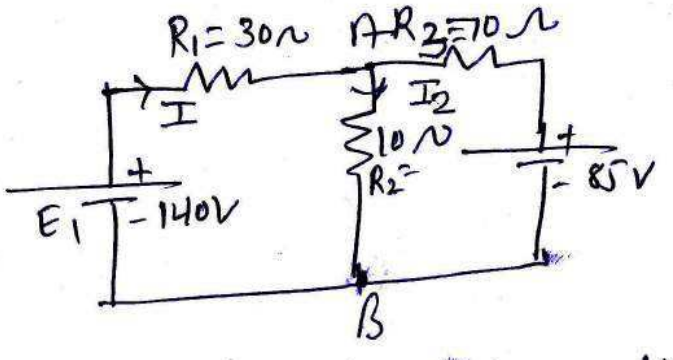


fig (1)

Ans
- I = 1 A , I_L = 1 A
- E_{th} = 12 V
- R_{th} = 6 Ω

(2) In the circuit given in figure (2), find the Branch current I_2 that flows through R_2 when R_2 has the the following values, 5Ω, 15Ω & 50Ω



Ans
- I = 0.55 A
- E_{th} = 123.5 V
- R_{th} = 21Ω
- R_2 = 5Ω , I_2 = 4.75 A

(3) In the circuit given in figure (3), find the current & voltage ~~at~~ in 2Ω Resistor using thevenin's theorem

Ans
- R_{th} = 8.75
- E_{th} = 27.5 V
- I_L = 2.56

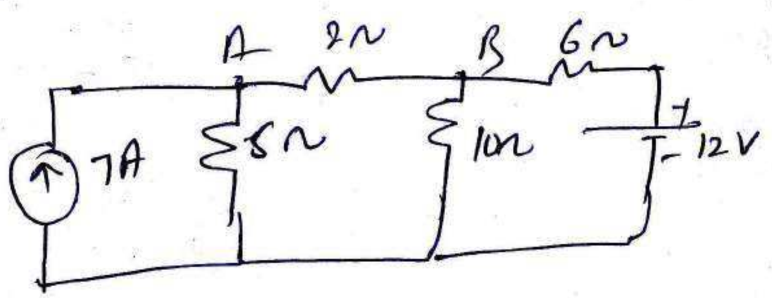


fig (3)

(4) Determine Current in 10Ω Resistor using Norton's theorem in the circuit shown in fig (4)

Ans ($I_1 = 1.288\ A$
$I_2 = I_N = 3.173$

$R_N = 7.4$

$I_L = 1.352$

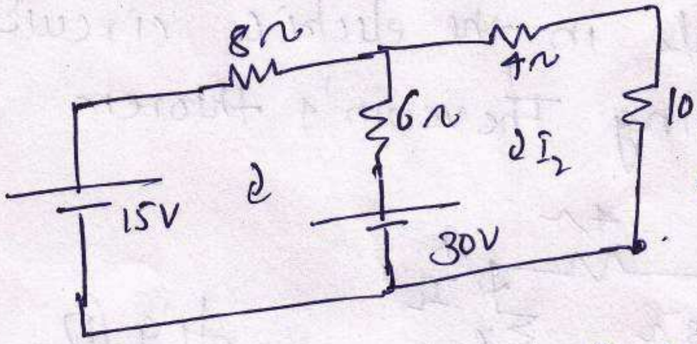
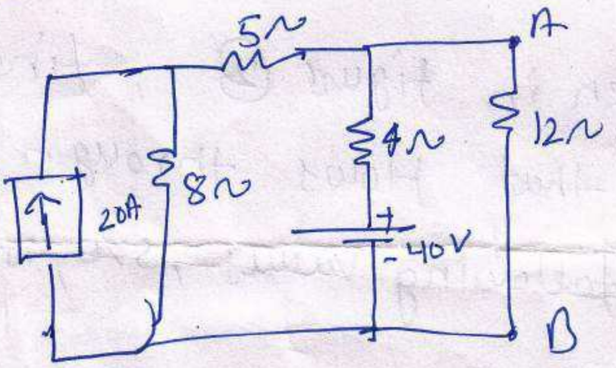


fig (4)

(5) Draw the Norton's equivalent circuit across AB and determine current flowing through 12Ω Resistor from the network shown in fig (5)



using superposition findout the I_{SC}

$I_{SC_1} = 20 \times \frac{8}{8+5} = 12.31\ A$

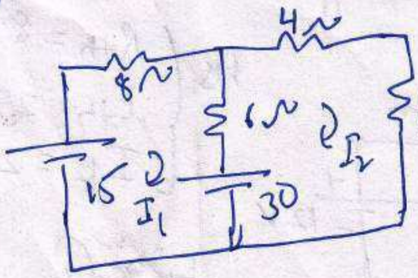
$I_{SC_2} = \frac{40}{4} = 10\ A$

$I_{SC} = I_{SC_1} + I_{SC_2}$

$R_N = \frac{(5+8) \times 4}{(5+8)+4} = 3.06\ \Omega = \frac{52}{17}$

$I_L = I_N \times \frac{R_N}{R_N + R_C} = 4.533\ A$

(6) Determine Current in 10-Ω Resistor using Norton's



Apply KVL for finding I_{SC}

$I_1 = .28$, $I_2 = 3.17 = I_N$

$R_N = \frac{8 \times 6}{8 \times 6} + 4 = 7.428$

$I_L = \frac{R_N}{R_N + R_L} \times I_N = 1.352\ A$

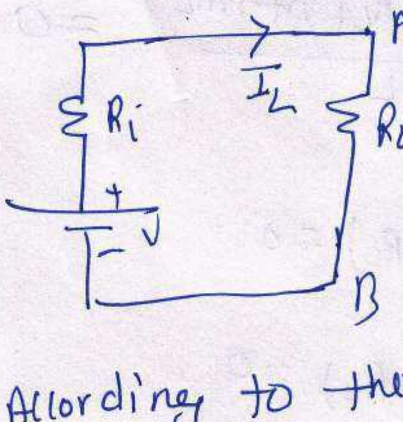
Maximum Power Transfer theorem:-

→ Suppose we have a Voltage source or battery that's Its internal resistance is R_i and a load Resistance is R_L is connected across the battery.

→ Maximum Power transfer theorem determines the value of Resistance R_L for which the maximum power will be transfered from source to it.

→ Actually the maximum power, drawn from the source, depends upon the value of the load Resistance

→ There may be some confusion let us clear it.

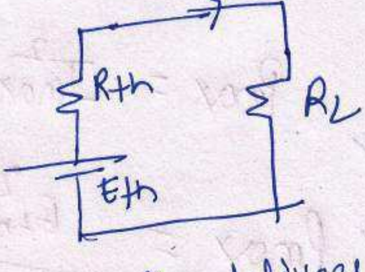


But According to thevenin theorem, every network can be represented by a single voltage source having an effective internal resistance R_{th}

So that

$$V = V_{th} = E_{th}$$

$$R_i = R_{th}$$



Let us determine the load Resistance R_L so that source delivers maximum Power

Power delivered to the load Resistance.

$$P = I_L^2 R_L = \frac{V^2}{(R_i + R_L)^2} R_L$$

Or

$$P = \frac{E_{th}^2}{(R_{th}+R_L)^2} \cdot R_L$$

To find the maximum power, differentiate the above expression with Respect to Resistance R_L & equate it to zero.

Thus

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left(\frac{E_{th}}{R_{th}+R_L}\right)^2 \cdot R_L = 0$$

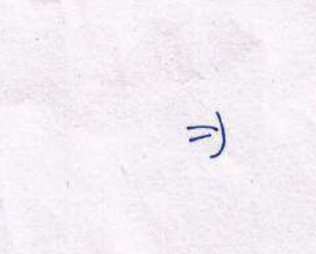
$$\Rightarrow E_{th}^2 \left(\frac{(R_{th}+R_L)^2 - 2R_L(R_{th}+R_L)}{(R_{th}+R_L)^2} \right) = 0$$

$$(R_{th}+R_L)^2 - 2R_L(R_{th}+R_L) = 0$$

$$(R_{th}+R_L)(R_{th}-R_L) = 0$$

$$R_{th} - R_L = 0$$

$$R_{th} = R_L$$



Load Resistance = Internal Resistance

$P_{max} = I_{max}^2 R_L$ (maximum Power delivered to the load)

$$P = I^2 R_L = \left(\frac{E_{th}}{R_{th}+R_L}\right)^2 \cdot R_L = \frac{E_{th}^2}{(R_L+R_L)^2} R_L$$

$$P_{max} = \frac{E_{th}^2}{4R_L}$$ (maximum Power transferred)

(34)

Q:- Find the value of load Resistance R_L for maximum Power flow through it in the circuit shown in fig-(34)

(34)

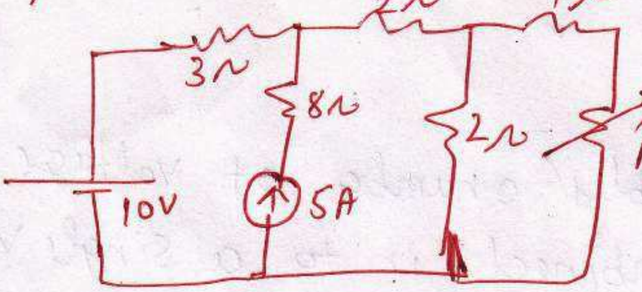
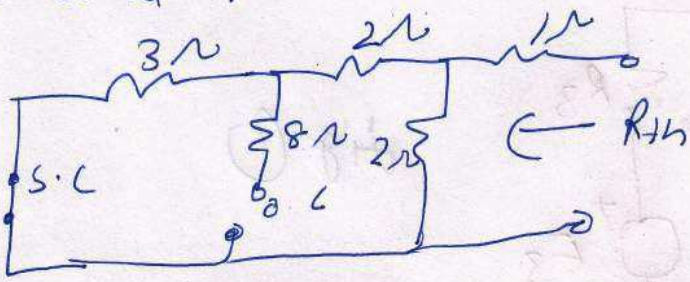


fig → (34)

Sol^n: The power drawn by the load Resistance R_L will be maximum when its value is equal to the R_{th}.

To find R_{th}, Replace voltage source S.C & current source open-circuit, as shown in fig-34-(1)



$$R_{th} = \frac{(3+2)\times 2}{3+2+2} + 1 = 2.43\Omega$$

the power transferred to the load by the source will be maximum, when

$$R_L = R_{th} = 2.43\Omega$$

Millman's theorem:-

→ This theorem is the combination of Thevenin's & Norton's theorem.

→ This theorem enables a number of voltage or current source to be combined in to a single voltage or current source.

→ Consider a network with two common terminals A & B ~~can be replaced by a single voltage~~ between which 3 voltage sources E_1, E_2 & E_3 of Internal Resistance R_1, R_2, R_3 are connected (fig ①)

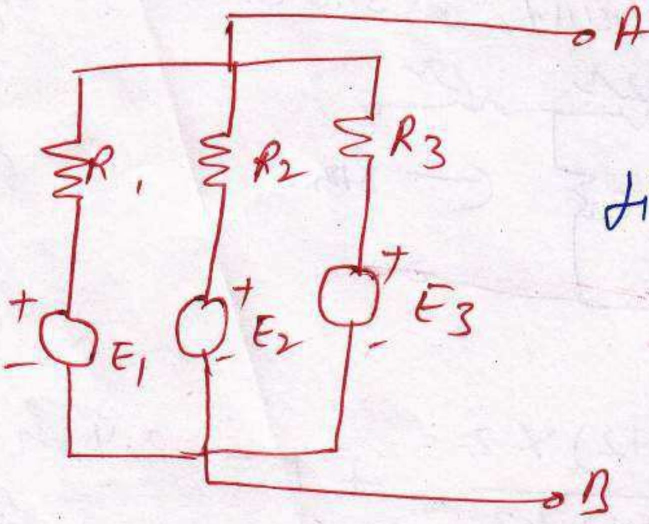


fig ①

→ According to this theorem, these voltage sources between terminals A & B can be Replaced by a single voltage source of V_{AB} volts in series with an equivalent Resistance R_{AB}

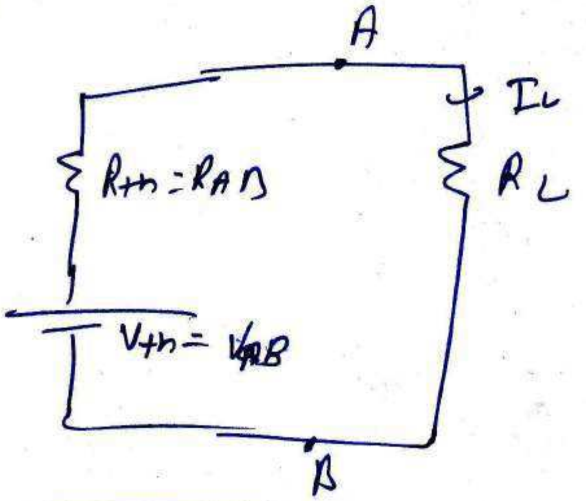
$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + \frac{1}{R_2} + 1/R_3} = \frac{E_1G_1 + E_2G_2 + E_3G_3}{G_1 + G_2 + G_3} = \frac{\Sigma EG}{\Sigma G}$$

($\because V = IR$)

$$R_{AB} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{G_1+G_2+G_3} = \frac{1}{\Sigma G}$$

voltage V_{AB} Represents the V_{th} &
R_{AB} Represents the R_{th}.

If there is a load Resistance R_L across the terminals A-B, then load current is given by

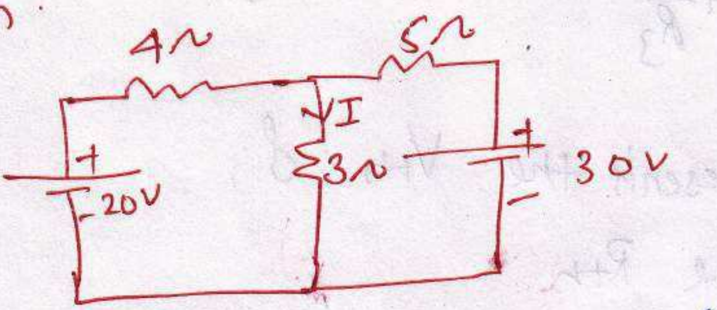


$$I_L = \frac{V_{th}}{R_{th}+R_L}$$

→ The above theorem is also applicable to current source.
→ According to this theorem any number of current source in parallel may be Replaced by a single current source.
→ If this theorem having combination of voltages and current source that are Reduced to a single constant current or constant voltage source.

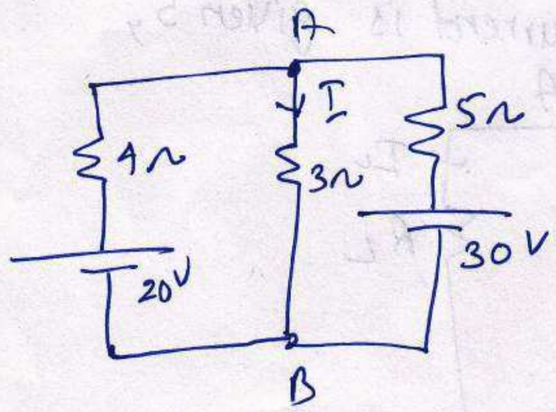
Numericals:-

① Calculate the current I shown in fig ① using Millman's theorem.



→ fig ①

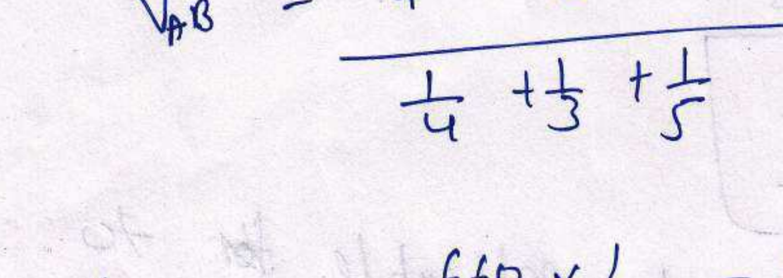
Sol^n: The circuit shown in fig ① can be Redrawn



$$V_{AB} = \frac{\frac{20}{4} + \frac{0}{3} + \frac{30}{5}}{\frac{1}{4} + \frac{1}{3} + \frac{1}{5}} = \frac{660}{47} \text{ V}$$

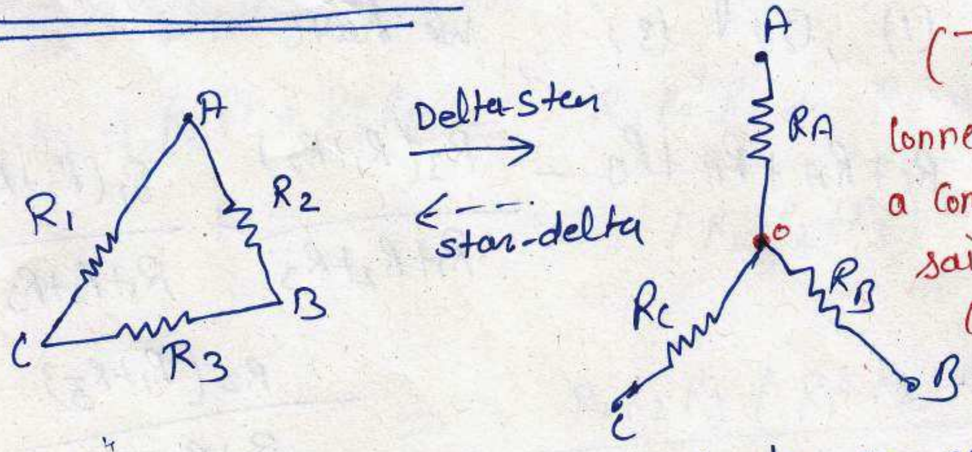
$$I = \frac{660}{47} \times \frac{1}{3} = \frac{220}{47} \text{ A}$$

② Determine the common terminal voltage across terminals A & B & the load current I_L in the n/w shown in fig ② using Millman's theorem.



Network Reduction By Delta-Star / Star-Delta Transformation

1) Delta-Star Transformation:-



(Three Resistances connected together at a common pt O, are said to be star- or (Y) connected)

The Replacement of delta or mesh by an equivalent star-system is known as delta-star transformation

Resistance b/w terminal B & C

$$R_{BC} = R_3 \| (R_1+R_2) = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} = \frac{R_1R_3+R_2R_3}{R_1+R_2+R_3}$$

in delta

$$R_{BC} = R_B + R_C \rightarrow \text{In star}$$

two systems are identical & Resistance measured b/w terminal A & C in both of the systems must be equal.

$$R_{BC} = R_B + R_C = \frac{R_3(R_1+R_2)}{R_1+R_2}$$

$$R_B + R_C = \frac{R_3(R_1+R_2)}{R_1+R_2} \quad - eq^n (1)$$

IIly

$$R_C + R_A = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \quad - eq^n (2)$$

$$R_A + R_B = \frac{R_2(R_1+R_3)}{R_1+R_2+R_3} \quad - eq^n \text{ (3)}$$

Adding eq^n (1), (2) & (3), we have

$$R_B+R_C + R_C+R_A + R_A+R_B = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} + \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} + \frac{R_2(R_1+R_3)}{R_1+R_2+R_3}$$

$$2R_B+2R_C+2R_A = \frac{R_1R_3 + R_2R_3 + R_1R_2 + R_1R_3 + R_2R_1 + R_2R_3}{R_1+R_2+R_3}$$

$$2(R_A+R_B+R_C) = \frac{2(R_1R_2+R_2R_3+R_3R_1)}{R_1+R_2+R_3}$$

$$R_A+R_B+R_C = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1+R_2+R_3} \quad - eq^n \text{ (4)}$$

subtracting eq^n (1), (2) & (3) from eq^n (4),

$$R_A = \frac{R_1R_2}{R_1+R_2+R_3} \quad \rightarrow eq^n \text{ (5)}$$

$$R_B = \frac{R_2R_3}{R_1+R_2+R_3} \quad \rightarrow eq^n \text{ (6)}$$

$$R_C = \frac{R_3R_1}{R_1+R_2+R_3} \quad \rightarrow eq^n \text{ (7)}$$

$R_1 = R_2 = R_3 = R_D$,

$$R_S = \frac{R_DR_D}{3R_D} = \frac{R_D}{3} \quad - eq^n \text{ (8)}$$ (star equivalent Resistance)

Star - Delta

Multiplying eq^n (5) & (6), (6) & (7), (7) & (5) & then adding

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2^2 R_3 + R_2 R_3^2 R_1 + R_3 R_1^2 R_2}{(R_1+R_2+R_3)^2} \quad \text{— eq}^n (9)$$

$$= \frac{R_1 R_2 R_3 (R_1+R_2+R_3)}{(R_1+R_2+R_3)^2}$$

~~$R_{AB} + R_{BC} + R$~~

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2 R_3}{R_1+R_2+R_3} \quad \text{— eq}^n (9)$$

Dividing eq^n (9) by eq^n (5), (6) & (7)

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_3 = R_B + R_C + \frac{R_B R_C}{R_A} \quad \text{— eq}^n (10)$$

$$R_1 = R_A + R_C + \frac{R_A R_C}{R_B} \quad \text{— eq}^n (11)$$

$$R_2 = R_A + R_B + \frac{R_A R_B}{R_C} \quad \text{→ eq}^n (12)$$

$$R_A = R_B = R_C = R_S$$

$$\boxed{R_D = 3 R_S}$$